Indian Institute of Technology Kharagpur
Course: MA51014/MA41004 Topology*
Spring Semester 2016
Assignment -03

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1. True or False. Justify your answer.
   (a) $\mathbb{R}$ with lower limit topology is a regular topological space.
   (b) A Hausdorff topological space is regular.
   (c) If (X, τ_x) and (Y, τ_y) are regular topological spaces then the product topological space $X \times Y$ is regular.
   (d) A metric space is regular topological space.
   (e) If $C_1, C_2, \ldots, C_n, n > 1$ are the only components of a topological space X then each $C_i, i = 1 : n$ is both open and closed subset of X.
   (f) Consider two topologies τ_1 and τ_2 are defined on a nonempty set X such that $\tau_1 \subset \tau_2$. Then (X, τ_1) is connected if and only if (X, τ_2) is connected.
   (g) Let $A_n, n \in \mathbb{N}$ be a connected subspace of topological space X such that $A_n \cap A_{n+1} \neq \emptyset$ for all n. Then $\cup_n A_n$ is connected.
   (h) Let X be a topological space and $A \subset X$. If C is connected subspace of X such that $C \cap A \neq \emptyset \neq C \cap (X \setminus A)$ then $C \cap \partial A \neq \emptyset$.
   (i) Product of path-connected spaces is path-connected.
   (j) If A is a path-connected subset of a topological space then $Cl(A)$ is also path-connected.

2. Let (X, d) be a metric space. Suppose $\rho : X \times X \to \mathbb{R}$ defined as $\rho(x, y) = \frac{d(x,y)}{1+d(x,y)}$. Then show that
   (a) (X, ρ) is a metric space.
   (b) The topologies on X induced by d and ρ are same.

3. Consider the set $\mathcal{B} = \{B_{p,q} : p, q \in \mathbb{Z}, q \neq 0\}$ of subsets of $\mathbb{Z}$, where

   $$B(p, q) = \{\ldots, p - 2q, p - q, p, p + q, p + 2q, \ldots\}.$$

   Show that
   (a) $\mathcal{B}$ is a basis for a topology in $\mathbb{Z}$.
   (b) The topological space $(\mathbb{Z}, \tau)$, where τ is generated by $\mathcal{B}$, is metrizable.

   Define a metric on $\mathbb{Z}$ that will induce the topology τ.

4. Let X be a topological space. Define a relation ρ on X as follows: $x\rho y$ if there is NO separation $X = A \sqcup B$ of X into disjoint open sets A and B such that $x \in A$ and $y \in B$. Then

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(a) Show this relation is an equivalence relation. (each class is called a quasicomponent of X).

(b) Determine the components, path components, and quasicomponents of the subspace $A \times [0,\ 1] \cup \{(0,0)\} \cup \{(0,1)\}$ of $\mathbb{R} \times \mathbb{R}$ where $A = \{1/n : n \in \mathbb{Z}_+\}$.