Indian Institute of Technology Kharagpur
Course: MA51014/MA41004 Topology*
Spring Semester 2016
Assignment -01

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1. True or False. Justify your answer.
   (a) The plane $\mathbb{R}^2$ with origin removed is homeomorphic to the plane with the closed unit circle/disc $\{(x, y) : x^2 + y^2 \leq 1\}$ removed.
   (b) The open unit disc/circle $\{(x, y) : x^2 + y^2 < 1\}$ is homeomorphic to any open disc in $\mathbb{R}^2$.
   (c) The unit sphere S^2 with north and south pole removed and the cylinder $\{(x, y, z) : x^2 + y^2 = 1, |z| < 1\}$ are homeomorphic.
   (d) The square $\{(x, y) : |x| \leq 1, |y| \leq 1\}$ and the closed unit disc in $\mathbb{R}^2$ are homeomorphic.
   (e) The torus is homeomorphic to a sphere.
2. Derive a formula of a homeomorphism from the open interval $(0,\ 1)$ to the real line $\mathbb{R}$.
3. Find at least ten examples of pairwise non-homeomorphic path connected sets in the plane such that each set is a union of three line segments and each such line segment is straight and homeomorphic to the open interval $(0,\ 1)$.
4. Consider the p-norm $(1 \leq p \leq \infty)$ equipped in the vector space $\mathbb{R}^2$. Is any unit open disc $\{x \in \mathbb{R}^2 : \|x\|_p < 1\}$ homeomorphic to $\{x \in \mathbb{R}^2 : \|x\|_q < 1\}$ when $p \neq q$?
5. Give an example of a path connected set in the plane that has infinitely many cut-points of type 1 and exactly one cut-point of type n for each $n \geq 2$.
6. Prove that the number of components is a topological property.
7. Prove that a homeomorphic image $f(S)$ of any subset S is homeomorphic to S.
8. Let S be a path-connected set. The pair (p, q) of two distinct points p amd q of S is called a cut-pair of type n if $S \setminus \{p, q\}$ has n components. Prove that the number of cut-pairs of any type is a topological property.
9. Construct examples of non-homeomorphic path connected sets in the plane where each pair of non-homeomorphic sets can be judged by the number of cut-pairs of type n for each n.
10. List down some characteristics/properties of a Möbius strip (that you have observed after having done some experiment as it was shown in the class).

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