MA 20205 Probability and Statistics
Hints/Solutions to Assignment 7

1. $p_{U,V}(0,1)=\frac{1}{12}, p_{U,V}(0,4)=\frac{1}{12}, p_{U,V}(1,1)=\frac{1}{3}, p_{U,V}(1,4)=\frac{1}{2}.$

2. $Z^2=(X_1-X_2)^2+(Y_1-Y_2)^2.$ $\frac{X_1-X_2}{\sqrt{2}}\sim N(0,1), \frac{Y_1-Y_2}{\sqrt{2}}\sim N(0,1).$ So $\frac{Z^2}{2}\sim\chi_2^2.$
Hence Z^2 has a negative exponential distribution with mean 4.

3. $f_{Y_1,Y_2}(y_1,y_2)=\lambda^2 y_2 e^{-\lambda y_2}.\frac{1}{(1+y_1)^2}, y_1>0, y_2>0.$ The densities of Y_1 and Y_2 can be derived easily now.

4. $f_{Y_1,Y_2}(y_1,y_2)=\frac{1}{2}e^{-y_1/2}.\frac{1}{\pi(1+y_2)^2}, y_1>0, y_2\in\mathbb{R}.$ Clearly Y_1 and Y_2 are independent.

5. Similar. Y and Z are independent. Z and U are independent.

6. Similar. $Y_{1,}$ Y_2 , …, Y_n are independent.

7. $\ln W = 2 + 2 \ln X_1 + 1.5 \ln X_2 + 1.28 \ln X_3 \sim N(17.06, 7.0692)$. Now use the symmetry of normal distribution to get the median of W and values of L and R.

8. $\rho = 0.5$, $P(-1 < X < 1|Y = 1) = 0.6772$, $V(2X + 3Y) = 19$, $P(-5 < 2X + 3Y < 8) = 0.842$.

9. Use the linearity property of normal distribution.
Reqd. Prob = 0.9544.