MA 20205 Probability and Statistics
Hints/Solutions to Assignment No. 5

1. The range of $Y = y$ is interval $[1, 4)$. For $0 < y < 1$, there are two inverse images; $x = \sqrt{y}$ and $x = -\sqrt{y}$. So for $0 < y \leq 1$,

$$f_Y(y) = \frac{2(\sqrt{y}+1)}{9}\frac{1}{2\sqrt{y}} + \frac{2(-\sqrt{y}+1)}{9}\frac{1}{2\sqrt{y}} = \frac{2}{9\sqrt{y}} \ .$$

For $1 < y < 4$, there is only one inverse image $x = \sqrt{y}$

So for $1 < y < 4$,

$$f_Y(y) = \frac{2(\sqrt{y}+1)}{9}\frac{1}{2\sqrt{y}} = \frac{1}{9}\left(1 + \frac{1}{\sqrt{y}}\right).$$

2. The range of $Y = y$ is interval $\left[0, \frac{9}{4}\right)$. There are two inverse images; $x = \frac{3}{2} + \sqrt{y}$ and $x = \frac{3}{2} - \sqrt{y}$. So for $0 < y \leq \frac{1}{4}$, we get

$$f_Y(y) = \frac{1}{2}\frac{1}{2\sqrt{y}} + \frac{1}{2}\frac{1}{2\sqrt{y}} = \frac{1}{2\sqrt{y}}.$$

For $\frac{1}{4} < y < \frac{9}{4}$, we get

$$f_Y(y) = \frac{1}{2}\frac{1}{2\sqrt{y}}\left(\frac{3}{2} - \sqrt{y}\right) + \frac{1}{2}\frac{1}{2\sqrt{y}}\left(3 - \frac{3}{2} - \sqrt{y}\right).$$

So we have

$$f_Y(y) = \begin{cases} \dfrac{1}{2\sqrt{y}} & if \quad 0 \leq y \leq \dfrac{1}{4} \\ \dfrac{3}{4\sqrt{y}} - \dfrac{1}{2} & if \quad \dfrac{1}{4} < y < \dfrac{9}{4} \\ 0 & elsewhere \end{cases}$$

3. The range of $Y = y$ is interval $(0, 1]$. There are two inverse images; $x = \sin^{-1} y$ and $x = \pi - \sin^{-1} y$.

So we get

$$f_Y(y) = \frac{2}{\pi^2}\left(\frac{\sin^{-1} y}{\surd(1-y^2)} + \frac{\pi - \sin^{-1} y}{\surd(1-y^2)}\right) = \frac{2}{\pi\surd(1-y^2)} \quad ,0 < y \le 1.$$

4. Apply direct pmf approach.
5. Apply pdf approach.
6. Apply pdf approach.
7. The range of $Y = y$ is $(0, \infty)$. For $y > 0$, there are two inverse images $x = \sqrt{\frac{2y}{m}}$ and $x = -\sqrt{\frac{2y}{m}}$. So we have

$$f_Y(y) = c\,\frac{2y}{m}\; e^{-\frac{2by}{m}}\,\frac{1}{\sqrt{2my}} + c\,\frac{2y}{m}\; e^{-\frac{2by}{m}}\,\frac{1}{\sqrt{2my}}.$$

After simplification

$$f_Y(y) = \frac{2c}{m}.\sqrt{\frac{2y}{m}}\;\; e^{-\frac{2by}{m}} \;, \qquad y > 0.$$

8. The range of $Y = y$ is interval $[0, 3)$. For $0 < y \le 1$, there are two inverse images; $x = y$ and $x = -y$. So for $0 < y \le 1$,

$$f_Y(y) = \frac{1+y}{4} + \frac{1-y}{4} = \frac{1}{2}$$

For $1 < y < 3$, there is only one inverse image $x = y$

So for $1 < y < 3$,

$$f_Y(y) = \frac{3-y}{4}.$$

9. The range of $Y = y$ is $(-\infty, \infty)$. For $y > 0, x = 4y^2$ and for $y < 0, x = -y^2$. So the pdf of Y is

$$f_Y(y) = \begin{cases} -\frac{2y}{\sqrt{2\pi}} e^{-\frac{y^4}{2}}, & y < 0 \\ x & \\ \frac{8y}{\sqrt{2\pi}} e^{-8y^4}, & y \ge 0 \end{cases}.$$

10. Apply direct pmf approach.

11. The range of $Y = y$ is $(0, 1)$. For $0 < y < 1$, we have $x = y^4$.

The pdf of X is $f_X(x) = \begin{cases} \frac{3}{2}\sqrt{x} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$.

So the pdf of Y is $f_Y(y) = \begin{cases} 6y^5 & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$

12. Apply pdf approach.