Problem Set - 6

AUTUMN 2016

MATHEMATICS-I (MA10001)

August 29, 2016

1. Expand $f(x,y) = x^2y + \sin y + e^x$ in power of $(x-1)$ and $(y-\pi)$ through quadratic terms and write the remainder.

2. Let $f(x,y) = e^x \sin y$. Expand $f(x+h, y+k)$ in power of h and k and also find R_2.

3. Expand $z = \sin x \sin y$ in power of $(x - \frac{\pi}{4})$ and $(y - \frac{\pi}{4})$. Find the terms of first and second orders and R_2.

4. Show that for $0 < \theta < 1$

$$e^{ax} \sin by = by + abxy + \frac{1}{6}[(a^3x^3 - 3ab^2xy^2)\sin(b\theta y) + (3a^2bx^2y - b^3y^3)\cos(b\theta y)]e^{a\theta x}.$$

5. Expand $x^3 - 2xy^2$ in Taylor's theorem about $a = 1, b = -1$.

6. Find the local maximum and minimum of $f(x,y) = x^4 + y^4 - x^2 - y^2 + 1$.

7. Find the local maximum and minimum of $f(x,y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$.

8. Show that the minimum value of $u = xy + \frac{a^3}{x} + \frac{a^3}{y}$ is $3a^2$.

9. Show that the function $f(x,y) = 2x^4 - 3x^2y + y^2$ does not have a maximum or a minimum at $(0,0)$.

10. Show that the function $f(x,y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^5$ has neither a maximum nor a minimum at $(0,0)$.

11. Find the stationary points of the function $z = x^3 + y^3 - 3xy$ and investigate their character.

12. Find the points of extremum of the function $z = 2xy - 3x^2 - 2y^2 + 10$.

13. Of all triangles with the same perimeter, determine the triangle with greatest area.

14. Prove that the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ is $\frac{8abc}{3\sqrt{3}}$.

15. Find the greatest and least values of the function $f(x,y) = xy$ that the function takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

16. Using Lagrange's multipliers method show that the maximum and minimum value of $ax+by$ (where $a, b > 0$ are constants) subject to constraint $x^2 + y^2 = 1$ are $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$ respectively.