Problem Set - 5

AUTUMN 2016

MATHEMATICS-I (MA10001)

August 22, 2016

1. If $z = \frac{x^2y^2}{x+y}$, then prove that $x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial x \partial y} = 2\frac{\partial z}{\partial x}$.

2. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u.$$

3. Let $H(x, y)$ be a homogeneous function of x and y of degree n having continuous first order partial derivatives and $u(x, y) = (x^2 + y^2)^{-\frac{n}{2}}$, then show that $x\frac{\partial}{\partial x}(Hu) + y\frac{\partial}{\partial y}(Hu) = 0$.

4. Is the converse of Euler's theorem for three variables true? Justify your answer.

5. If $u = \sin^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$.

6. Let $f(x, y)$ and $g(x, y)$ be two homogeneous functions of degree m and n respectively, where $m \neq 0$ and $h = f + g$. If $x\frac{\partial h}{\partial x} + y\frac{\partial h}{\partial y} = 0$, then show that $f = \alpha g$ for some scalar α.

7. If $x^y + y^x = c$, then find $\frac{d^2y}{dx^2}$.

8. If $w = f\left(\frac{y-x}{xy}, \frac{z-y}{zy}\right)$, then show that

$$x^2\frac{\partial w}{\partial x} + y^2\frac{\partial w}{\partial y} + z^2\frac{\partial w}{\partial z} = 0.$$

9. If $u = f(r, s, t)$ where $r = \frac{x}{y}$, $s = \frac{y}{z}$ and $t = \frac{z}{x}$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0.$$

10. Let f be any differentiable function. Then show that $w(x, y) = f(xy^2)$ satisfies

$$2x\frac{\partial w}{\partial x} - y\frac{\partial w}{\partial y} = 0.$$

11. Consider the function $f(x, y) = x^2y^3 - 2y^2$. Check whether $f_{xy} = f_{yx}$ or not.

12. Find the t-derivative of $z = f(x(t), y(t))$, where $f(x, y) = x^5y^6$, $x(t) = e^t$ and $y(t) = \sqrt{t}$.

13. What is the t-derivative of $z = f(x(t), y(t))$ at $t = 1$ if $x(1) = 2$, $y(1) = 3$, $x^{'}(1) = -4$, $y^{'}(1) = 5$, $f_x(2, 3) = -6$ and $f_y(2, 3) = 7$?

14. Find $G^{'}(2)$ where $G(t) = h(t^2, t^3)$ and $h = h(x, y)$ is such that $h_x(4, 8) = 10$ and $h_y(4, 8) = -20$.

15. Find g_{yxx} for $g(x, y) = x^4 \sin(3y) + 5x - 6y$.

16. What are the s and t-derivatives of $z = f(st^2, te^s)$ at $s = 0$, $t = 2$ if the derivatives of $z = f(x, y)$ have the values $f_x(0, 2) = 10$ and $f_y(0, 2) = -5$?