INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

PROBLEM SET- 4 **AUTUMN 2016**

MATHEMATICS-I(MA10001) **2nd August 2016**

1. Find the total differential of
   (i) $u = \tan(3x - y) + 6^{y+z}$ (ii) $w = \sin^{-1}(\frac{x}{y})$ (iii) $z = \ln(xy)$ (iv) $z = e^{x^2+y^2}$.

2. Find the total derivatives of the given functions:
   (i) $u = \frac{e^{ax}(y-z)}{a^2+1}, \ \ y = a\sin x, \ \ z = \cos x$ (ii) $z = \ln(1-x^4), \ \ x = \sqrt{\sin\theta}$
   (iii) $u = x^2 - 2y^2 + z^3, \ \ x = \sin t, \ \ y = e^t, \ \ z = 3t$ (iv) $z = \ln(x^2+y^2), \ \ x = e^{-t}, \ \ y = e^t$.

3. If $z = f(x,y) = xy^2 + x^2y$ and $y = \ln x,$ find $\frac{dz}{dx}$ and $\frac{dz}{dy}$.

4. Show that the function $z = \phi(x^2 - y^2)$, where $\phi(u)$ is a differentiable function, satisfies the relationship $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0.$

5. Show that the homogeneous differentiable function of zeroth order $z = F(\frac{y}{x})$ satisfies the relationship $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0.$

6. Find the derivatives $\frac{dy}{dx}$ of the functions represented implicitly
   (i) $\sin(xy) - e^{xy} - x^2y = 0$ (ii) $xe^y + ye^x - e^{xy} = 0$ (iii) $y^x = x^y$ (iv) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

7. If $r = x\phi(x+y) + y\psi(x+y),$ show that
$$\frac{\partial^2 r}{\partial x^2} - 2\frac{\partial^2 r}{\partial x\partial y} + \frac{\partial^2 r}{\partial y^2} = 0.$$
   (ϕ and ψ are twice differentiable function.)

8. If $u = \frac{1}{y}[\phi(ax+y) + \phi(ax-y)]$, show that
$$\frac{\partial^2 u}{\partial x^2} = \frac{a^2}{y^2}.\frac{\partial}{\partial y}(y^2\frac{\partial u}{\partial y}).$$

9. Test the differentiability of the function $f(x,y) = \begin{cases} \frac{x^2-y^2}{x-y} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ at $(0,0)$.

10. Show that $f(x,y) = |x|(1+y)$ is continuous at $(0,0)$. Is it differentiable at $(0,0)$? Justify your answer.

11. Test the differentiability of the function $f(x,y) = \begin{cases} \frac{x^6-2y^4}{x^2+y^2} & x^2+y^2 \neq 0 \\ 0 & (x,y) = (0,0) \end{cases}$ at $(0,0)$.

Answers

Q. 1.

(i) $du = 3sec^2(3x-y)dx + (-sec^2(3x-y) + \log 6.6^{y+z})dy + (6^{y+z}\log 6)dz$

(ii) $dw = \frac{1}{\sqrt{y^2-x^2}}dx - \frac{x}{y\sqrt{y^2-x^2}}dy = \frac{ydx-xdy}{y\sqrt{y^2-x^2}}$

(iii) $dz = (\frac{1}{x})dx + (\frac{1}{y})dy$

(iv) $dz = 2e^{x^2+y^2}(xdx+ydy)$

Q. 2.

(i) $\frac{du}{dx} = e^{ax}\sin x$.

(ii) $\frac{dz}{d\theta} = -2\tan\theta$.

(iii) $\frac{du}{dt} = \sin 2t - 4e^{2t} + 81t^2$.

(iv) $\frac{2}{(e^{-2t}+e^{2t})}[e^{2t} - e^{-2t}]$.

Q. 3. $\frac{dz}{dx} = (\ln x)^2 + 2x(\ln x) + 2(\ln x) + x,\ \frac{dz}{dy} = y^2e^y + 2ye^{2y} + 2ye^y + e^{2y}$.

Q. 6.

(i) $\frac{dy}{dx} = \frac{y[-\cos(xy)+e^{xy}+2x]}{x[\cos(xy)-e^{xy}-x]}$.

(ii) $\frac{dy}{dx} = \frac{e^y+ye^x-ye^{xy}}{xe^{xy}-xe^y-e^x}$.

(iii) $\frac{dy}{dx} = \frac{yx^{y-1}-y^x\log y}{xy^{x-1}-x^y\log x}$.

(iv) $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$.

Q. 9.

The function $f(x,y)$ is differentiable at (0,0).

Q. 10.

Not differentiable at(0,0).

Q. 11.

The function $f(x,y)$ is differentiable at (0,0).

End