Problem Set - 2

AUTUMN 2016

MATHEMATICS-I (MA10001)

July 25, 2016

1. Determine the following limits:

   (a) $\dfrac{e^x - e^{-x} - x}{x^2 \sin x}$; $x \to 0$

   (b) $(2x \tan x - \pi \sec x)$; $x \to \pi/2$

   (c) $\cos(ax)^{b/x^2}$; $x \to 0$

   (d) $(\sin x)^{\tan^2 x}$; $x \to \pi/2$

   (e) $(1 - x^2)^{1/\ln(1-x)}$; $x \to 1$

   (f) $\dfrac{a^{\sin x} - a}{\ln \sin x}$; $x \to \pi/2$

   (g) $(\sec x)^{\cot x}$; $x \to \pi/2$

   (h) $\dfrac{1 - 4\sin^2(\pi x/6)}{1 - x^2}$; $x \to 1$

   (i) $\dfrac{(1+x)^{1/x} - e}{x}$; $x \to 0$

   (j) $\dfrac{\log_{sec(\frac{x}{2})} \cos x}{\log_{\sec x} \cos \frac{x}{2}}$; $x \to 0$

   (k) $\left(\dfrac{1}{x}\right)^{\tan x}$; $x \to 0$

   (l) $\dfrac{x^2 \sin(1/x)}{\tan x}$; $x \to 0$

2. Using L'Hospital rule evaluate

   (a) $\lim\limits_{x\to 0} (\ln \cot x)^{\tan x}$

   (b) $\lim\limits_{x\to\infty} \dfrac{e^{1/x^2} - 1}{2\arctan x^2 - \pi}$

   (c) $\lim\limits_{x\to 0} x^{\frac{1}{\ln(e^x)-1}}$

   (d) $\lim\limits_{x\to 0} \dfrac{sinx}{\sqrt{1 - cosx}}$

   (e) $\lim\limits_{x\to 1} [\dfrac{x}{x-1} - \dfrac{1}{\ln x}]$

   (f) $\lim\limits_{x\to 0} (\cot x)^{\frac{1}{\ln x}}$

   (g) $\lim\limits_{x\to 0} \left(\dfrac{1}{x}^{\tan x}\right)$

   (h) $\lim\limits_{x\to\infty} \left(1 + \dfrac{a}{x}\right)^x$

   (i) $\lim\limits_{x\to\pi/2} (\cos x)^{\frac{\pi}{2} - x}$

   (j) $\lim\limits_{x\to 1} (\tan \dfrac{\pi x}{4})^{tan \frac{\pi x}{2}}$

3. Expand in power of $x - 2$ of the polynomial $x^4 - 5x^3 + 5x^2 + x + 2$.

4. Expand in power of $x + 1$ of the polynomial $x^5 + 2x^4 - x^2 + x + 1$.

5. Write Taylor's formula for the function $y = \sqrt{x}$ when $a = 1, n = 3$.

6. Write the Maclaurin formula for the function $y = \sqrt{1+x}$ when $n = 2$. Further, estimate the error of the approximate equation $\sqrt{1+x} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2$ when $x = 0.2$.

7. Write down the Taylor's expansion for the function $f(x) = sinx$ about the point $a = \frac{\pi}{4}$ with $n = 4$.

8. If $f''(x)$ exists on $[a, b]$ and $f'(a) = f'(b)$ prove that

$$f\left(\frac{a+b}{2}\right) = \frac{1}{2}[f(a) + f(b)] + (b-a)^2 f''(c) \text{ for some } c \in (a, b).$$

9. Applying Taylor's theorem with remainder prove that $1 + \frac{x}{2} - \frac{x^2}{8} < \sqrt{1+x} < 1 + \frac{x}{2}$ if $x > 0$.

10. Applying Maclaurin's theorem with the remainder to expand

    (a) $\ln(1+x)$ (b) $(1+x)^m$

11. Using Taylor's formula, evaluate

    (a) $\lim\limits_{x\to 0} \dfrac{x - \sin x}{e^x - 1 - x - \frac{x^2}{2}}$ (b) $\lim\limits_{x\to 0} \dfrac{2(\tan x - \sin x) - x^3}{x^5}$

    (c) $\lim\limits_{x\to 0}[x - x^2 \ln(1 + \dfrac{1}{x})]$ (d) $\lim\limits_{x\to 0}(\dfrac{1}{x^2} - \dfrac{\cot x}{x})$

Note: Submit solution of ***problems 1,3,4,7*** to your tutorial teacher next week.