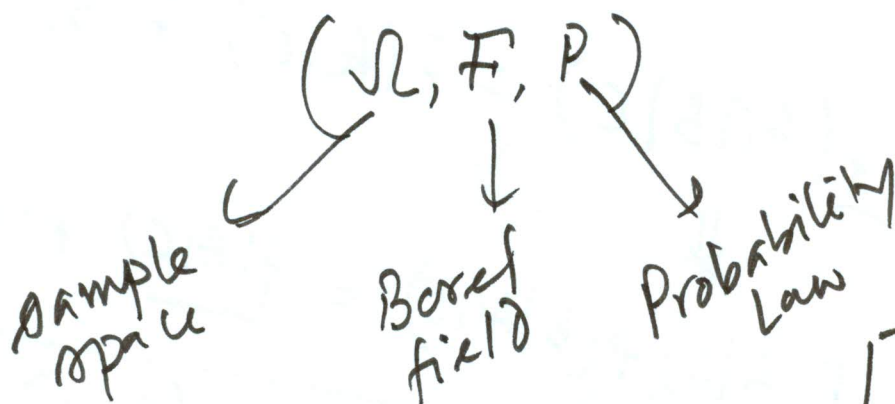


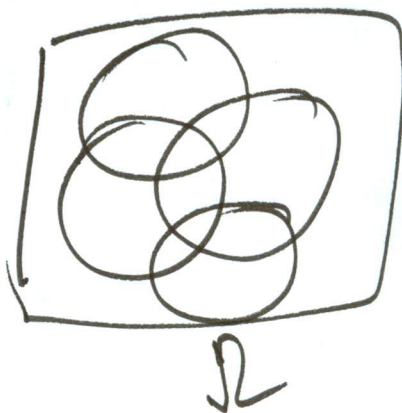
Lecture - 6

Probability & Statistics.

Independence of events.



$$P(A \cap B) = P(A) P(B)$$



$$A_1, A_2, \dots, A_n, \quad n \geq 2$$

$$P\left(\bigcap_{i \in S} A_i\right) = \prod_{i \in S} P(A_i)$$

where S is any subset of $\{1, \dots, n\}$

$n=3$. $P(A_i \cap A_j) = P(A_i) P(A_j), \quad i \neq j$

$\rightarrow$ pairwise independent

$$P\left(\bigcap_{i=1}^n A_i\right) = \prod_{i=1}^n P(A_i) \rightarrow \text{mutual independence}$$

$$P(A \cap B | \Omega) = P(A | \Omega) P(B | \Omega)$$

Conditional independence

$$P(A \cap B | C) = P(A | C) P(B | C)$$

$\Downarrow$

$$P(A | C) P(B | A \cap C) = \underline{P(A | C)} P(B | C)$$

$$\Rightarrow \boxed{P(B | A \cap C) = P(B | C)} \quad \checkmark$$

$$\underline{P(B | C)} P(A | B \cap C) = P(A | C) \underline{P(B | C)}$$

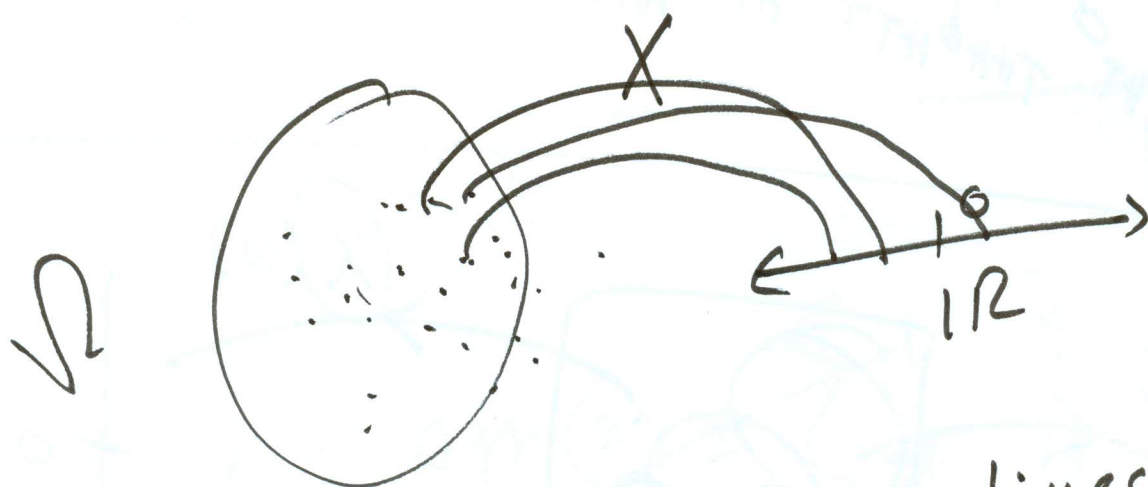
$$\Rightarrow \boxed{P(A | B \cap C) = P(A | C)}$$

independence $\nRightarrow$ cond. independence

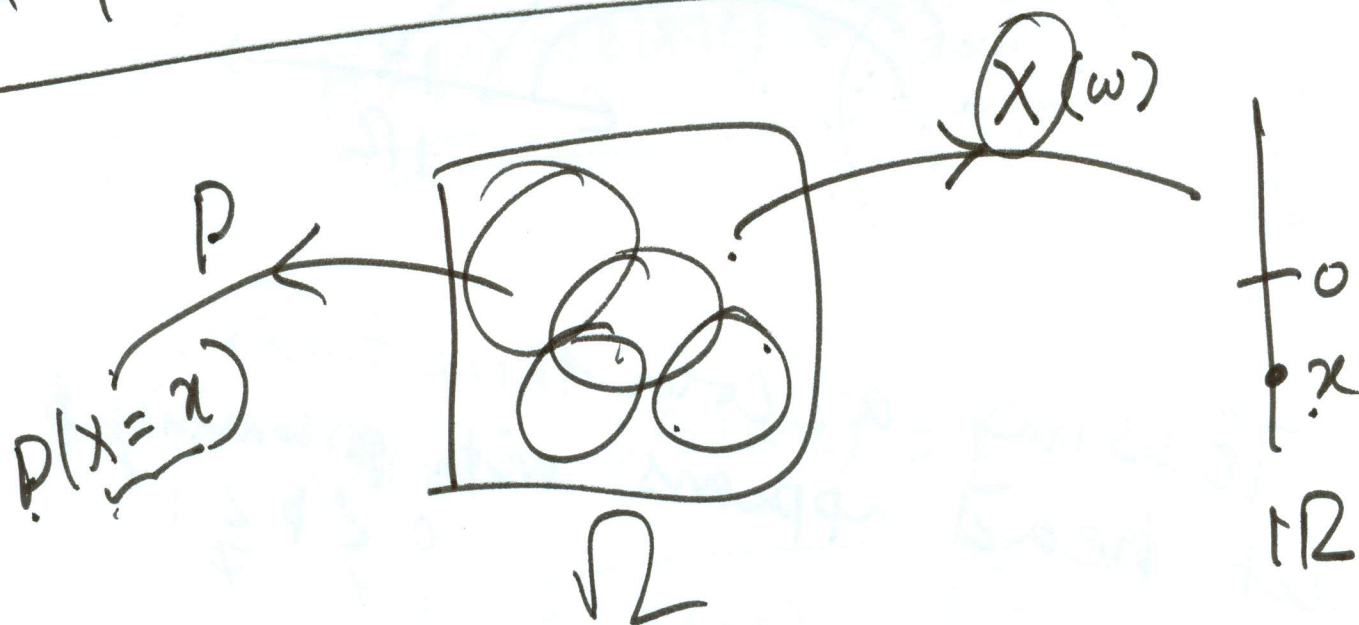
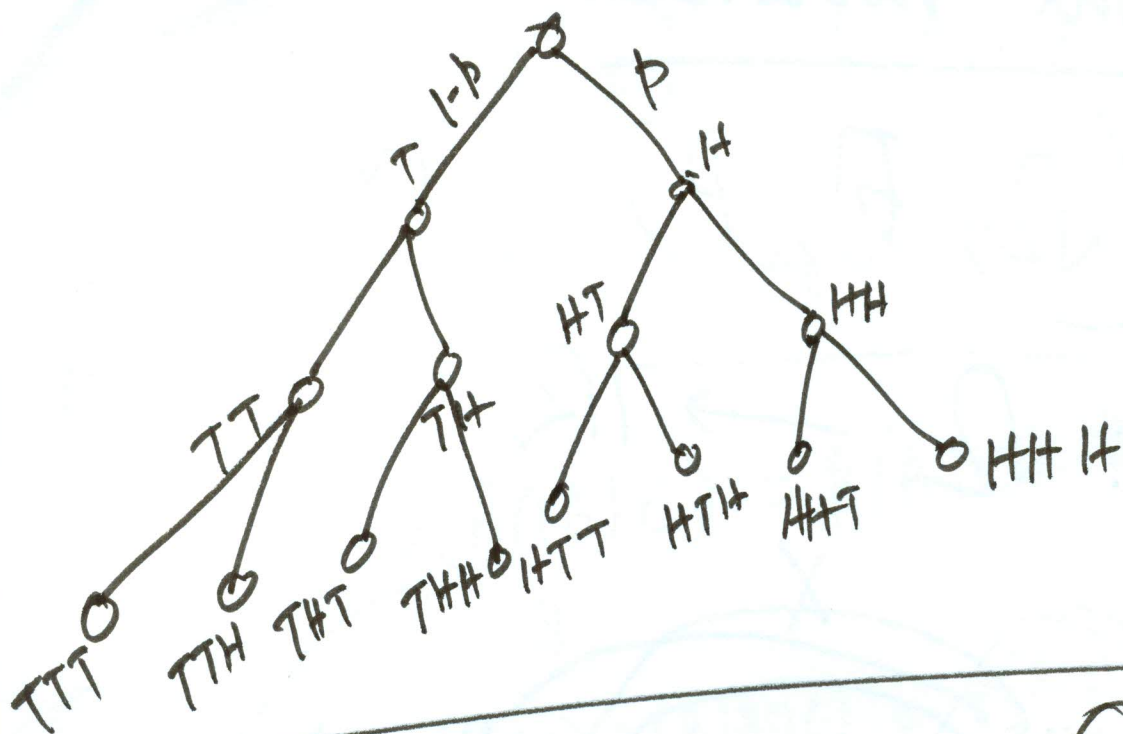
Random variable.

$$(\Omega, \mathcal{F}, P) \rightarrow \text{rv}$$

$$X: \Omega \rightarrow \mathbb{R}$$



Tossing a coin three times.
let head appears with probability p .
 $0 \leq p \leq 1$



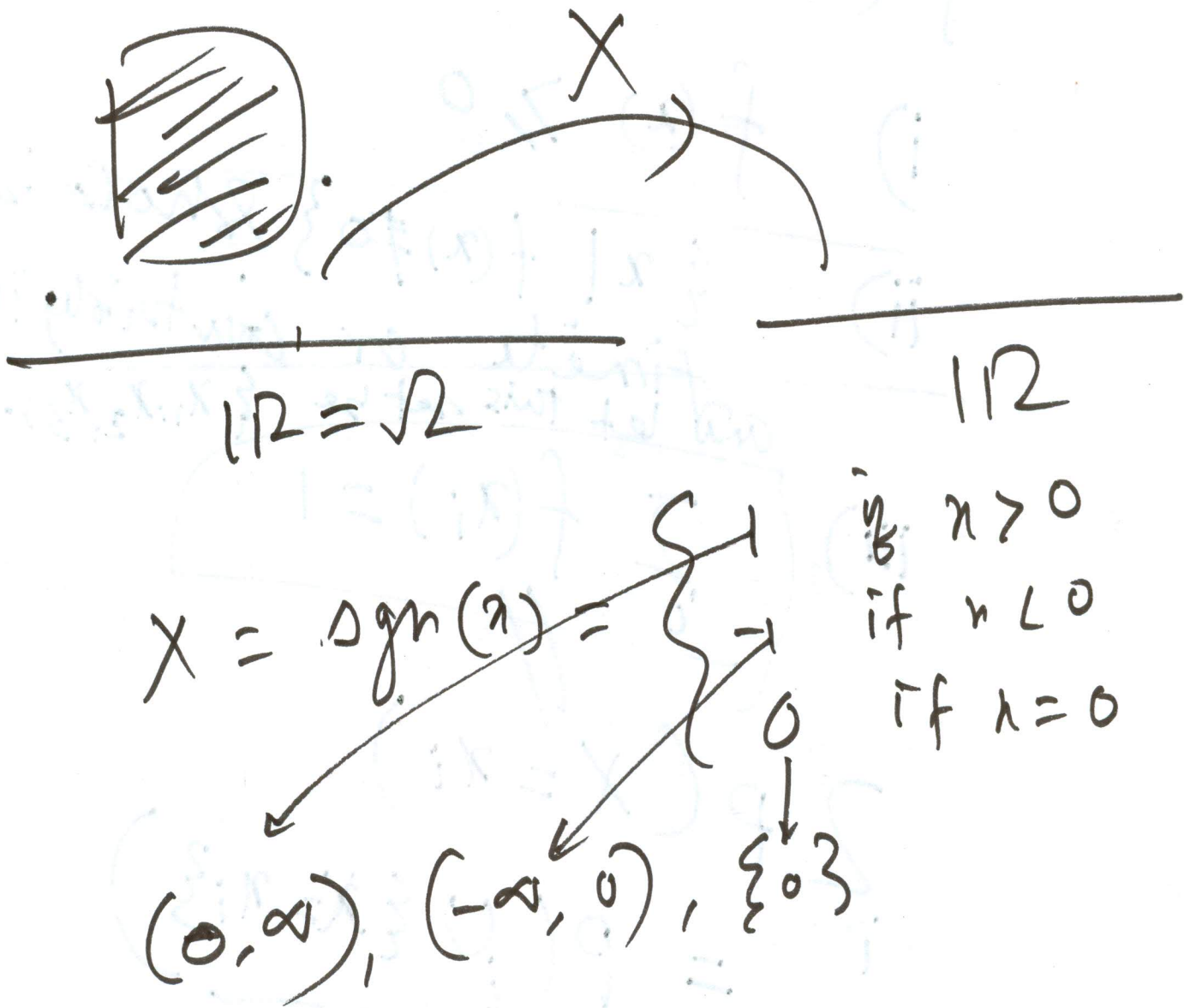
PMF

$$f: \mathbb{R} \rightarrow \cancel{[0, 1]} + \mathbb{R}$$

f

Many things are discussed on black board as the projector was not working for sometime

Discrete RV



PMF/PDF

$X, f: \mathbb{R} \rightarrow \mathbb{R}$

$$f(x) = P(X=x), x \in \mathbb{R}$$

i) $f(x) \geq 0$

ii) $\{x \mid f(x) \neq 0\}$ should be finite or countably inf.

and let this set be $\{x_1, x_2, x_3, \dots\}$

iii) $\sum_i f(x_i) = 1$

$$\begin{aligned} \sum_i P(X=x_i) \\ &= P(\cup_i \{X=x_i\}) \\ &= P(\Omega) = 1 \end{aligned}$$

Let $f: \mathbb{R} \rightarrow [0, 1]$

and suppose $\{x_1, x_2, \dots\}$ is
the set for which $f(x_i) \neq 0$.

Consider

$$\Omega = \{x_1, x_2, \dots\}$$

$$\mathcal{F} = \underline{\text{all subsets of } \Omega}$$

$$(\Omega, \mathcal{F}, P) \quad P(\{\omega\}) = f(x_i) \text{ if } \omega = x_i$$

Then define the random
variable $X: \Omega \rightarrow \mathbb{R}$ as

$$X(\omega) = x_i \quad \forall \omega = x_i$$

$$\{\omega \mid X(\omega) = x_i\} = \{x_i\} \text{ and thus}$$
$$P(X = x_i) = P(\{x_i\}) = f(x_i)$$

~~Instead of~~

we simply introduce a
random variable X
taking values $x_1, x_2, \dots$

such that $X = x_i$ if and

only if the experiment
results in the i th outcome.

