# Lecture-4.

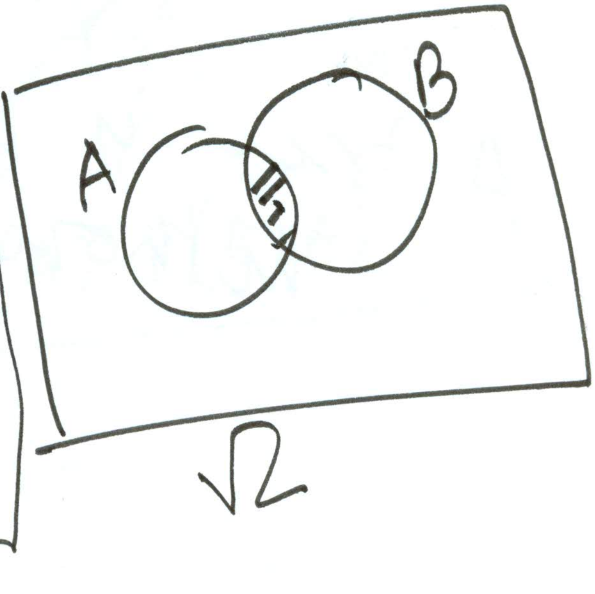
## Probability & Statistics.

### Conditional probability.

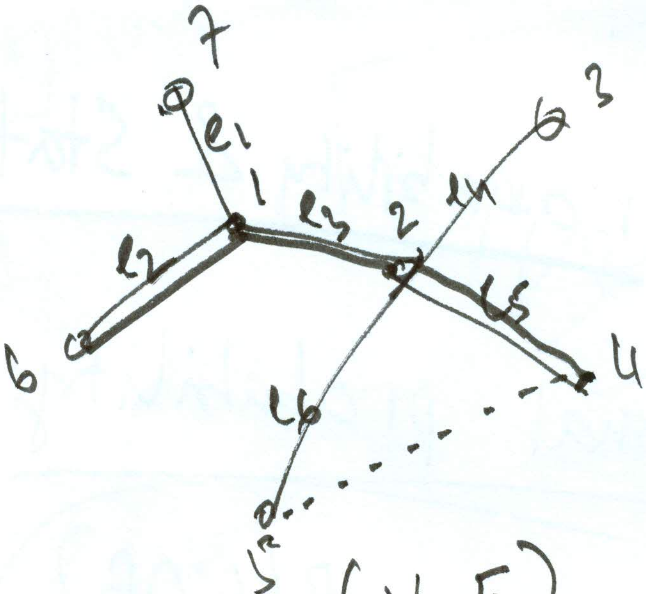
$$P(A|B) = \frac{P(A\cap B)}{P(B)}, \quad P(B) > 0$$

useful: when calculating prob. of an event A knowing that the event B has happened.

$$\boxed{\begin{aligned} P(A\cap B) &= P(B)\,P(A|B) \\ &= P(A)\,P(B|A) \end{aligned}}$$




$\Omega$



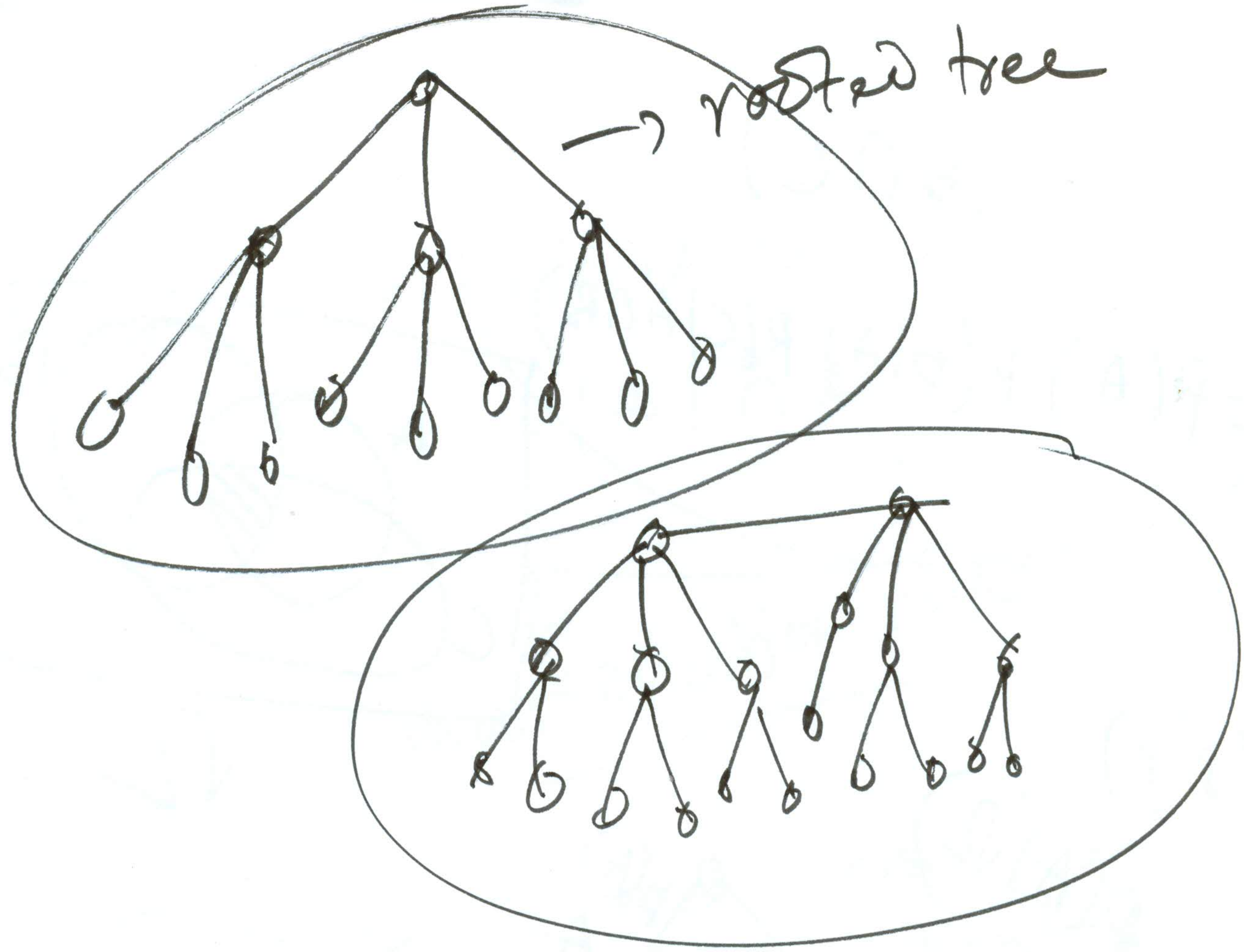

$$G = (V, E)$$

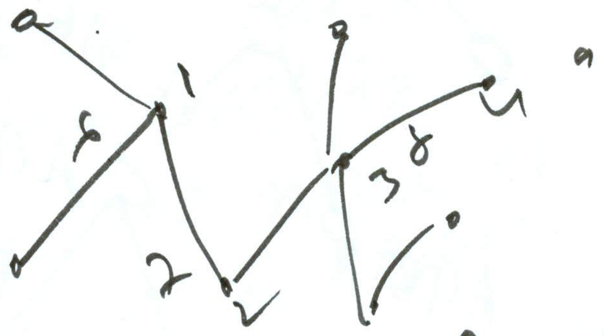
$$V = \{1, \dots, 7\}$$

$$E \subseteq V \times V$$

$$(6,4)$$

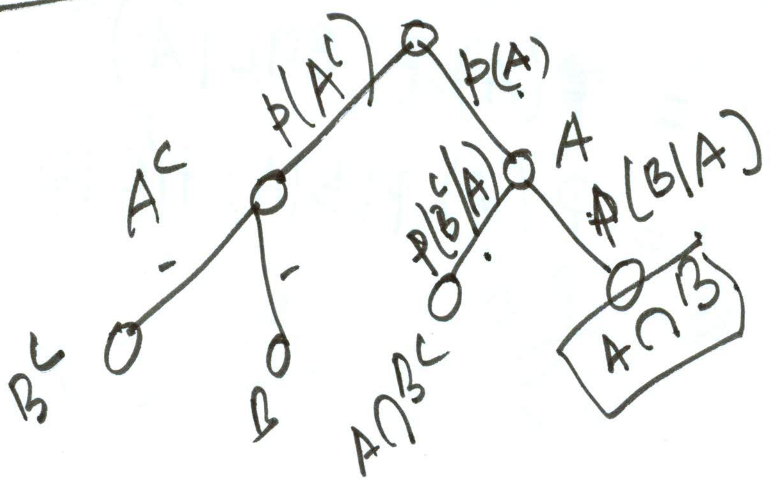
A tree is a connected graph without cycle.



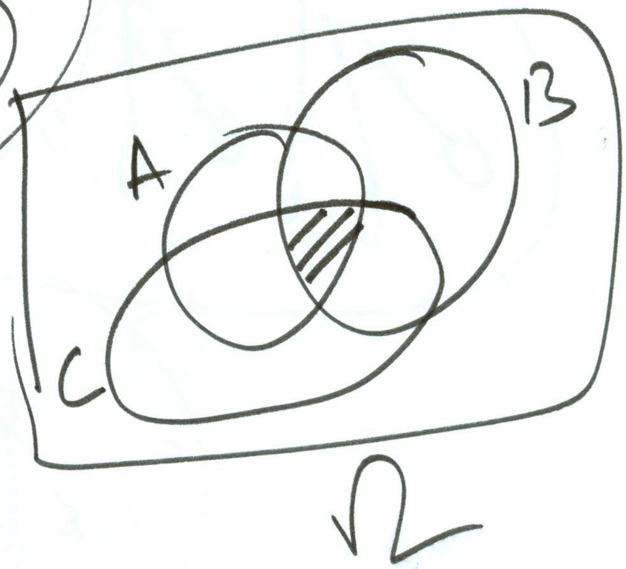




$n = 100$

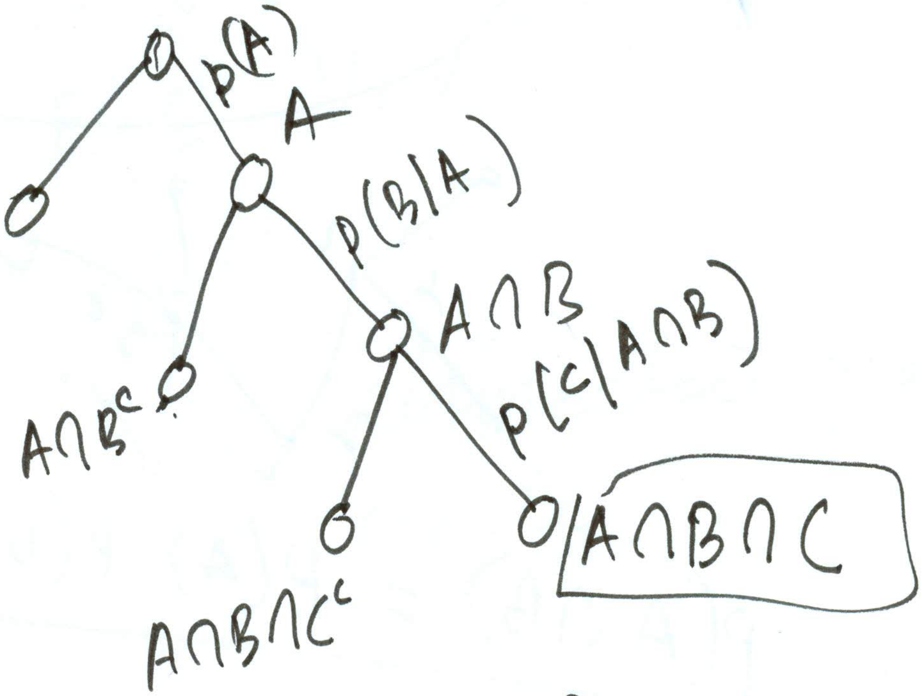
$$P(A \cap B) = P(A)\, P(B|A)$$



$$P(A\cap B\cap C) = P(A)\,P(B|A)\,P(C|A\cap B)$$




$$P(A) = P(A|\Omega)$$




$$P(A\cap[\underbrace{B\cap C}]) = P(A)\,P(B\cap C|A)$$
$$= P(A)\,P(B|A)\,P(C|B\cap A)$$

$$P(A_1 \cap A_2 \cap \cdots \cap A_n)$$

$$= P(A_1)\, P(A_2 | A_1)\, P(A_3 | A_1 \cap A_2) \cdots \cdots P(A_n | \bigcap_{i=1}^{n-1} A_i)$$

↓

<u>multiplication rule</u>.

<u>Prob1</u>. Three cards are drawn from an ordinary 52-card deck without replacement. We wish to find the probability that none of the three cards is a heart.

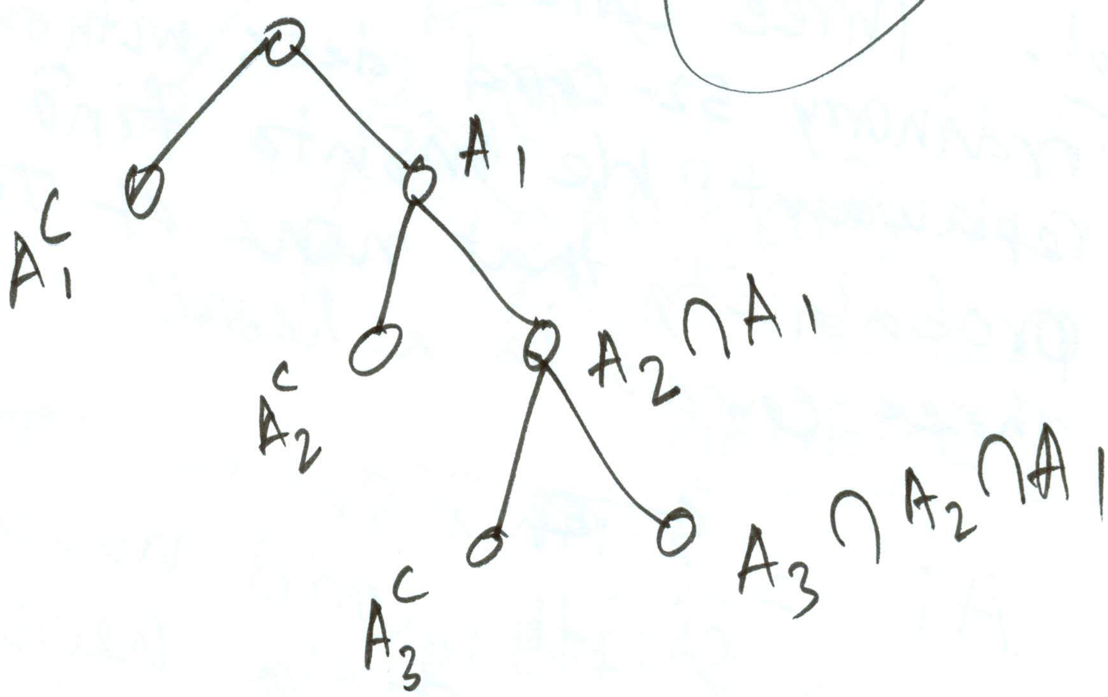
$A_i$ = { ~~ith~~ ~~x | outcom ith~~ ith card which is not a heart }, $i = 1, 2, 3$.

$$P(A_1) = 1 - \frac{13}{52} = \frac{39}{52}$$

$$P(A_1 \cap A_2 \cap A_3)$$

$$= P(A_1)\, P(A_2 \mid A_1)\, P(A_3 \mid A_1 \cap A_2)$$

$$\frac{39}{52} \times \frac{38}{51} \times \frac{37}{50}$$



Prob. 2 A class consisting of 4 AG students and 12 MI students is randomly divided into 4 groups of 4. What is the probability that each group includes an AG student?

By random we mean that given the assignment of some students to certain slots, any of the remaining students is equally likely to be assigned to any of the remaining slots.

$$\frac{12}{15} \times \frac{8}{14} \times \frac{4}{13}$$

Let 1, 2, 3, 4 be the AG students.

$A_1 = \{$ 1 & 2 are in different groups $\}$

$A_2 = \{$ 1, 2 & 3 . . . . . . $\}$

$A_3 = \{$ 1, 2, 3, & 4, . . . . $\}$

$P(A_3) = P(A_1 \cap A_2 \cap A_3)$

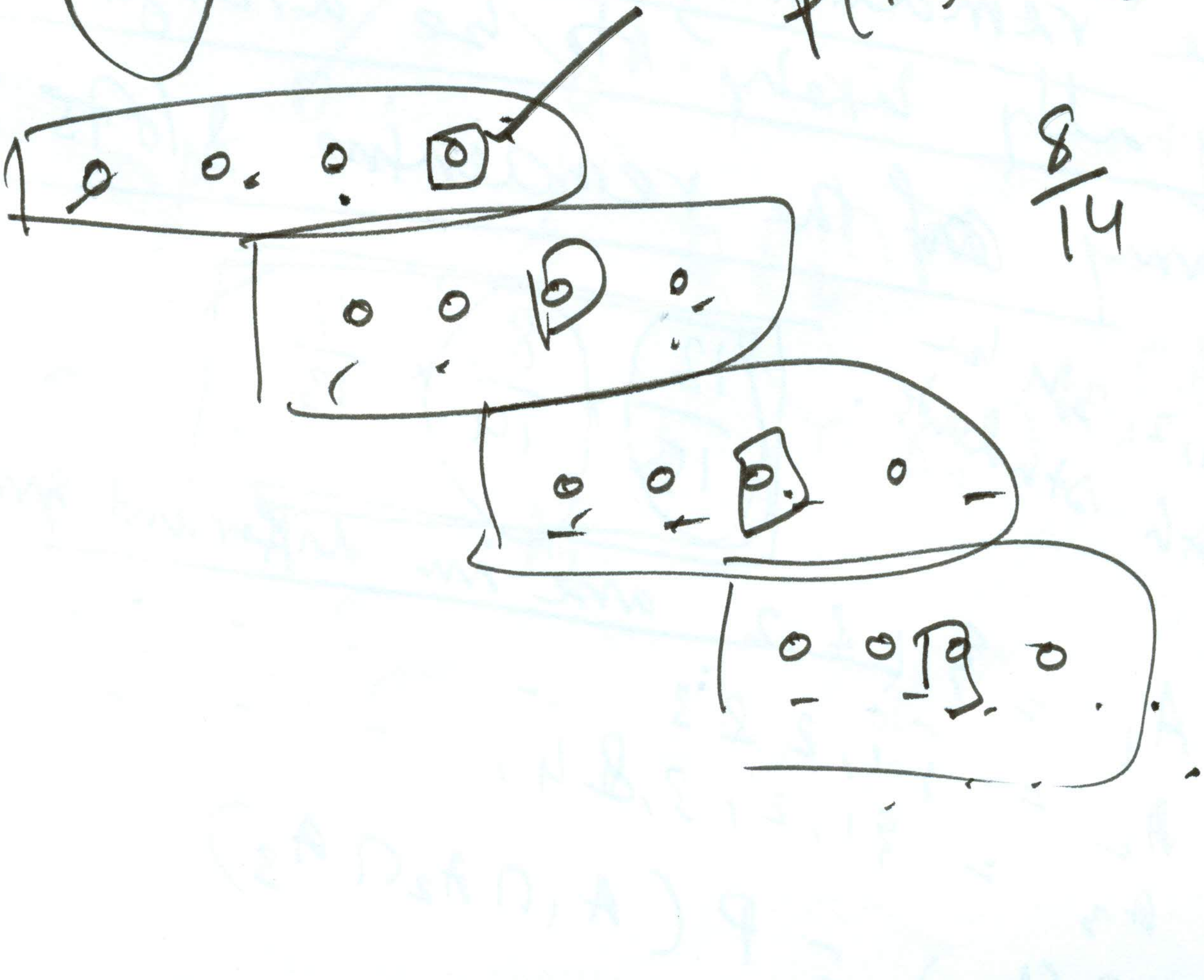
$$= P(A_1)\, P(A_2 | A_1)\, P(A_3 | A_1 \cap A_2)$$

$$= \frac{12}{15} \times \frac{P(A_2 \cap A_1)}{P(A_1)} \times \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cap A_2)}$$

1 Ah AG

$$\frac{12}{15} \times \frac{8}{14} \times \frac{4}{13}$$

$$P(A_1) = \frac{12}{15}$$



$$\frac{8}{14}$$

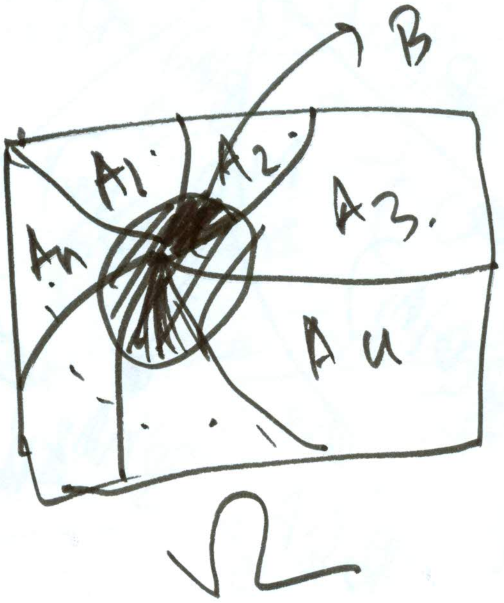
# Applications of cond'l prob.

Assume that

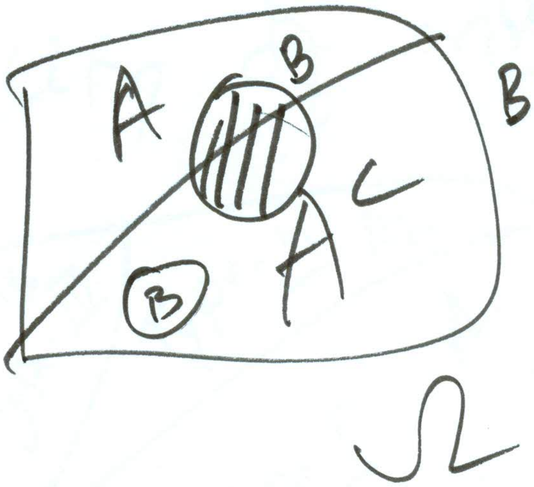
$A_i,\ i=1:n$

form a partition for

$\Omega$




$\boxed{P(A_i)} \neq 0$




$B = (A \cap B) \cup (A^c \cap B)$

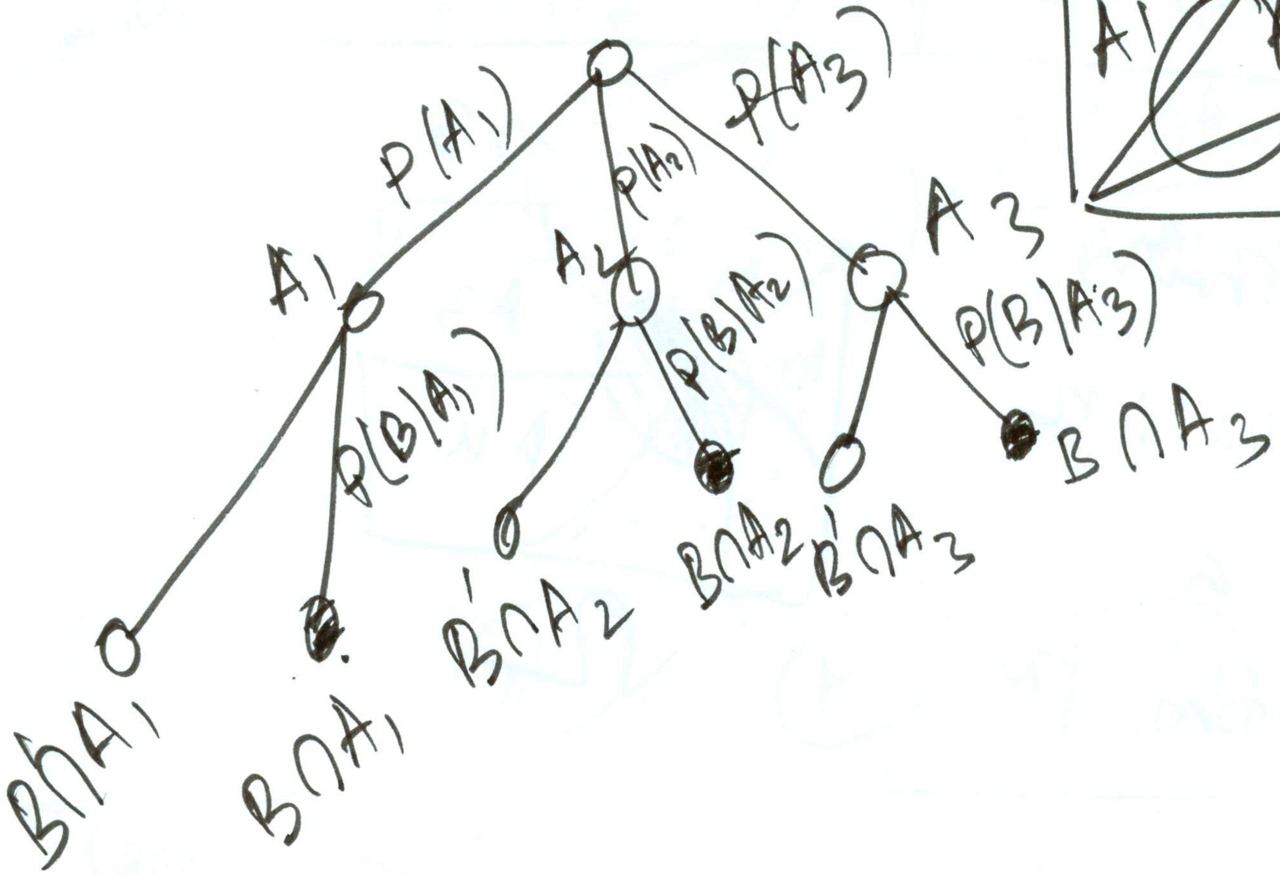
Use $P(A)$ & $P(A^c)$ to compute $P(B)$

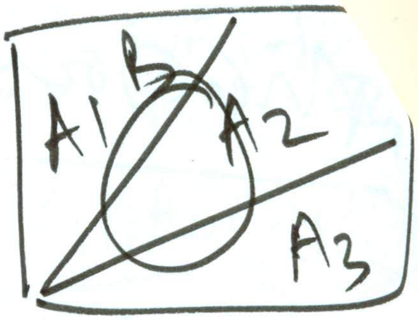
$$B = (A_1 \cap B) \cup (A_2 \cap B) \cup \dots \cup (A_n \cap B)$$

$$\cancel{P(B)} \quad P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \dots + P(A_n \cap B)$$

$$= P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \dots + P(A_n)P(B|A_n)$$

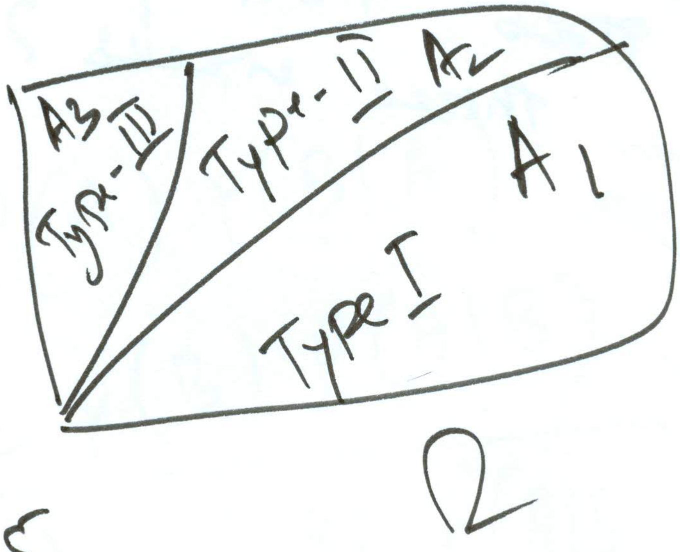
Total probability theorem


P(A1)
P(A2)
P(A3)
A1
A2
A3
P(B|A1)
P(B|A2)
P(B|A3)
B'∩A1
B∩A1
B'∩A2
B∩A2
B'∩A3
B∩A3


B
A1
A2
A3

Prob. You enter a Chess tournament where your probability of winning a game is 0.3 against half of the players (call them Type I), 0.4 against a quarter of the players (call them Type-II), and 0.5 against the remaining quarter of players (call them Type-III).

You play a game against a randomly chosen opponent. What is the probability of winning.




$P(A_1) = \frac{1}{2} = .5$

$P(A_2) = .25$

$P(A_3) = 0.25$

$P(B|A_1) = 0.3, \quad P(B|A_2) = 0.4$

$P(B|A_3) = 0.5$

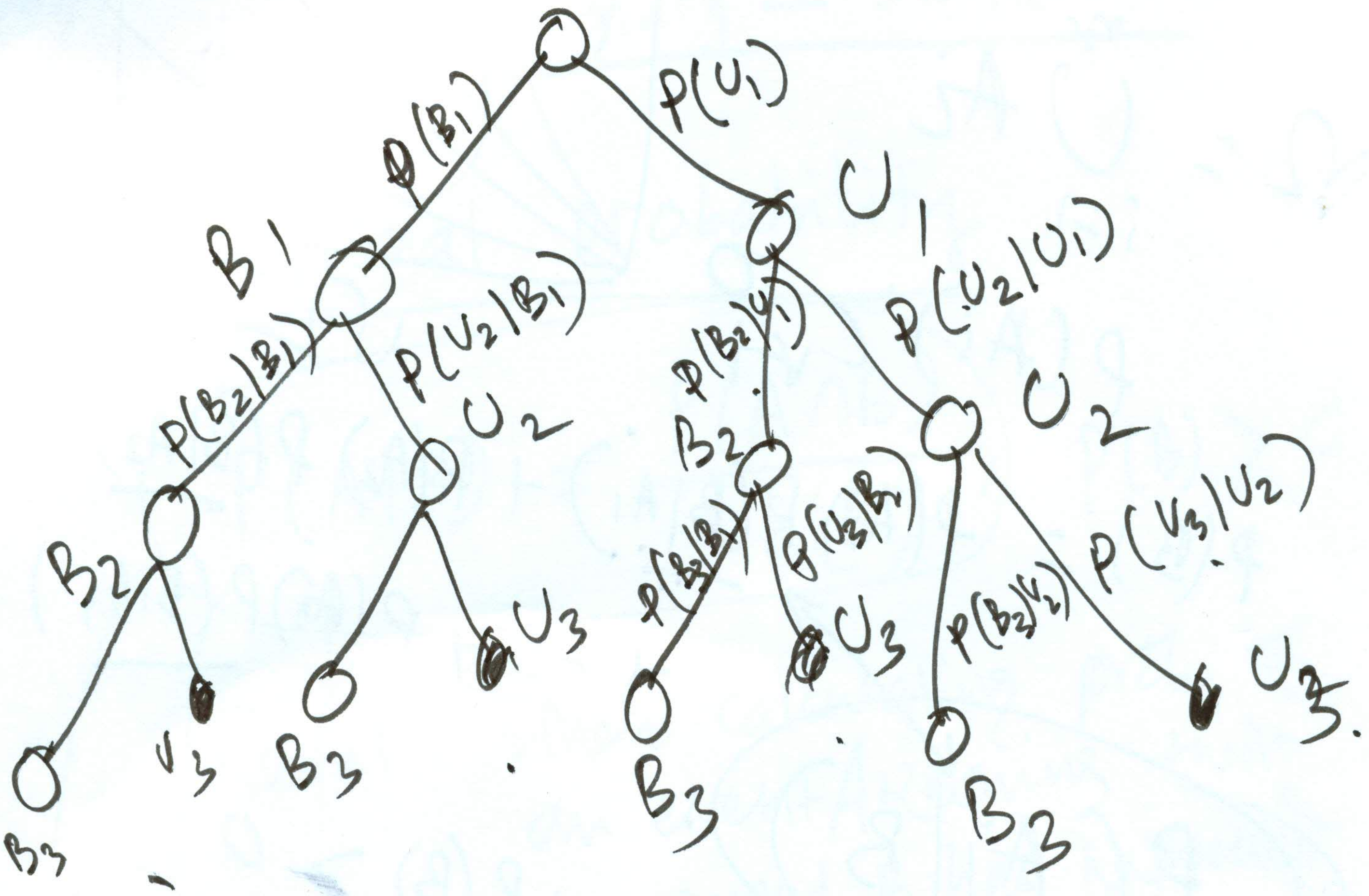
$P(B) = ? \quad 0.375$

**Prob.** You are taking this Prob & Stat. course and at the end of each week you can be either up-to-date or you may have fallen behind.

If you are up-to-date in a given week, the prob. that you will be up-to-date (or behind) in the next week is 0.8 (or 0.2 respectively).

If you are behind in a given week, the prob. that you will be up-to-date (or behind) in the next week is 0.4 (or 0.6 resp.).

You are (by default) up-to-date when you start the course. What is the prob that you are up-to-date after three weeks?

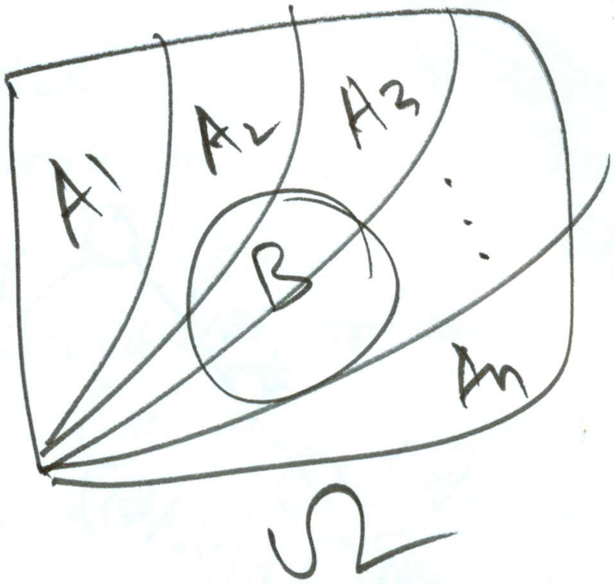



$$P(A \wedge B) = P(A)\, P(B|A)$$
$$= P(B)\, P(A|B)$$

$$\boxed{P(A)\, P(B|A) = P(B)\, P(A|B)}$$

$A_i$'s are disjoint

$$\Omega = \bigcup_{i=1}^{n} A_i$$




$$P(A_i) > 0 \quad \forall i$$

$$P(B) = P(A_1)\,P(B|A_1) + P(A_2)\,P(B|A_2) + \cdots + P(A_n)\,P(B|A_n)$$

$$P(B) > 0$$

Baye's Rule

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i)\,P(B|A_i)}{P(B)}$$

$$= \frac{P(A_i)\,P(B|A_i)}{P(A_1)\,P(B|A_1) + P(A_2)\,P(B|A_2) + \cdots + P(A_n)\,P(B|A_n)}$$