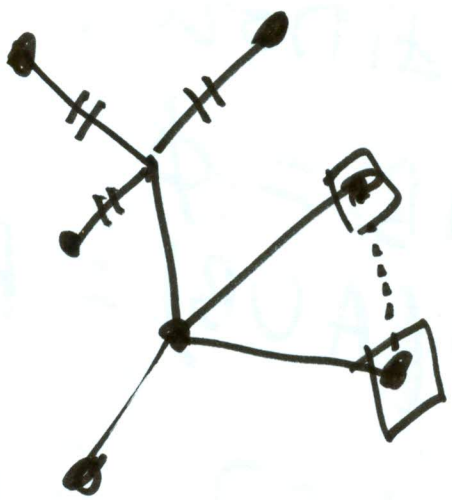
Lecture - 3

Probability & Statistics.

Uncertain Situation.

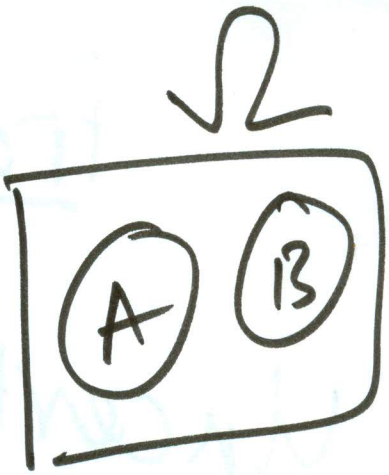


Probability Model.

- sample space.
- Events — subsets of the sample space
- Probability law

$P: \mathcal{F} \longrightarrow [0, 1]$

$\downarrow$ set of events



1) $P(A) \geq 0$

2) Addition:

$A \cap B = \phi$

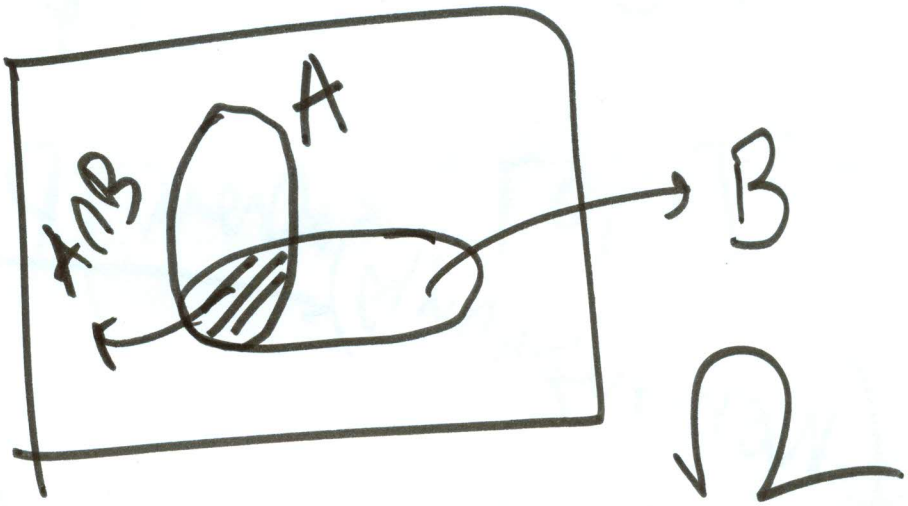
$P(A \cup B) = P(A) + P(B)$

3) $P(\Omega) = 1$

$(\Omega, \mathcal{F}, P)$

$\mathcal{F}$: Borel field

Conditional Probability.



$P(A) = ?$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \qquad P(B) > 0$$

$$P(A \cap B) = P(B)\, P(A|B)$$

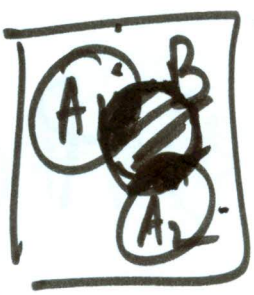
$$= P(A)\, P(B|A)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

when $P(B) > 0$ (non-negativity)

(i) $P(A|B) \geq 0$?

(ii) if $A_1 \cap A_2 = \phi$



$$P(A_1 \cup A_2 | B)$$
$$= P(A_1|B) + P(A_2|B)$$

CHS. $P(A_1 \cup A_2|B) = \frac{P([A_1 \cup A_2] \cap B)}{P(B)}$

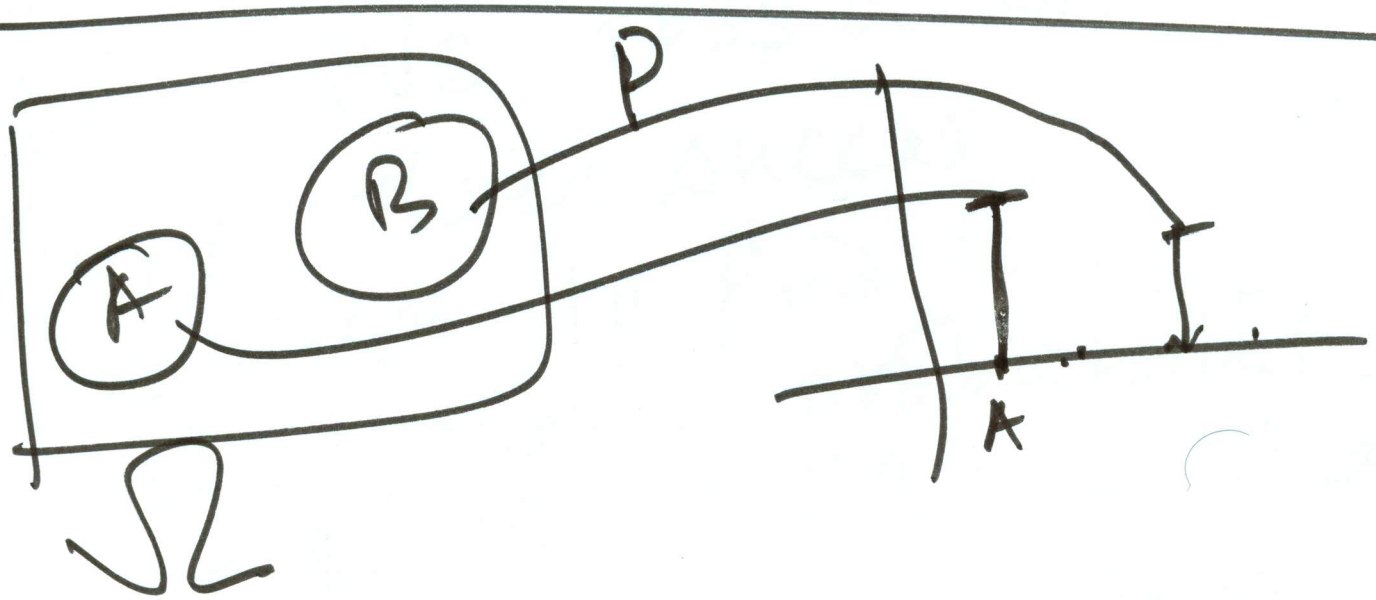
$$= \frac{P([A_1 \cap B] \cup [A_2 \cap B])}{P(B)} = \frac{P(A_1 \cap B) + P(A_2 \cap B)}{P(B)}$$
$$= P(A_1|B) + P(A_2|B)$$

$$M_{2\times 2} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \right\}$$

$$P : F \longrightarrow [0, 1]$$

$$P(A) = \det(A) \quad \cancel{P(A) = 1 - P(A^c)}$$

$$\det(A \pm B) = \det(A) + P(B)$$



3) $P(\Omega|B) = 1$

$$P(\Omega|B) = \frac{P(\Omega \cap B)}{P(B)}$$

$$= \frac{P(B)}{P(B)}$$

$$\Omega = \{H,T\}^3 \quad = 1.$$

Exp1: We toss a fair coin three successive times. We wish to find the conditional probability $P(A|B)$ where A & B are the events: $A = \{$ more heads than tails come up$\}$

$B = \{$1st toss is a head$\}$.

$$\Omega = \{(i,j,k) \mid i,j,k \in \{H,T\}\},$$

Ans. $P(A|B) = 3/4$

Prb 2- A fair 4-sided die is rolled twice and we assume that all sixteen possible outcomes are equally likely. Let X & Y be the results of the 1st & the 2nd roll respectively. We wish to determine the cond. prob. $P(A|B)$

$$A = \{\max(X,Y) = m\}$$

$$B = \{\min(X,Y) = 2\}$$

$$m = 1, 2, 3, 4.$$

$m=1$ $\quad P(A|B)=0$

$m=2$ $\quad P(A|B)=\frac{1}{5}$

$m=3$ $\quad P(A|B)=\frac{2}{5}$

$m=4$ $\quad \vdots$

Bob 3 - A Conservative design team, call it C and an innovative design team, call it N, are asked to separately design a new product within a month. From past experience we know that

1) The prob. that C is successful is 2/3
2) The prob. that N is succ. is 1/2
3) The prob. that at least one of them is succ. is 3/4.

Q. Assume that exactly one successfull design is produced. what is the probability that it was team N?