Lecture-23

Probability & Statistics.

Estimate like ~~values~~ parameters (unknown) for a given population.

↓

interval

Normal population

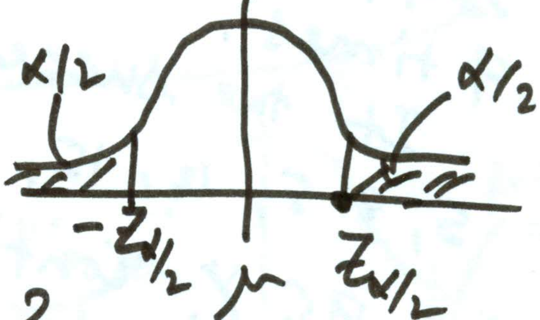
$\sigma^2 \rightarrow$ known

estimate μ (population mean)

Let $x_1, x_2, \ldots, x_n$

$$\bar{X} = \frac{\sum X_i}{n} \sim N(\mu, \sigma^2/n)$$

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$



$$P\left\{-z_{\alpha/2} < Z < z_{\alpha/2}\right\} = 1-\alpha$$

$$\Rightarrow P\left\{-z_{\alpha/2} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < z_{\alpha/2}\right\} = 1-\alpha$$

$$\Rightarrow P\left\{-z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu - \bar{X} < z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right\} = 1-\alpha$$

$$\Rightarrow P\left\{\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right\} = 1-\alpha$$

A $100(1-\alpha)$ percent confidence interval for μ is

$$\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

where $\bar{x}$ is the observed sample mean.

Exp. Suppose that when a signal having value μ is transmitted from location A the value received at location B is normally distributed with mean μ and variance 4. That is, if μ is sent, then the value $\mu + N$ where N ~~resp.~~ representing noise, is normal with mean 0 and variance 4. To reduce error, suppose that the same value is sent 9 times.

If the successive values received are 5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5, find a 95% confidence interval for μ.

Solⁿ. $\bar{x} = \frac{81}{9} = 9$

$$\left(9 - 1.96 \cdot \frac{\sigma}{3},\ 9 + 1.96 \cdot \frac{\sigma}{3}\right) = (7.69, 10.31)$$