Lecture - ~~20~~ 22

Probability & Statistics

Sampling distributions:

$\bar{X}, S^2, \chi^2_n, t_n$

$X_1, X_2, \ldots, X_n$, μ, σ^2

population mean: μ & variance: σ^2

$\bar{X} \sim N(\mu, \sigma^2/n)$.

$\chi^2_n = Z_1^2 + Z_2^2 + \cdots + Z_n^2$

$Z_i \sim N(0,1)$, Z_i are independent

$\chi^2_n \sim \text{Gamma}\left(\frac{n}{2}, \frac{1}{2}\right)$

normal population $\left\{ \boxed{(n-1)\frac{S^2}{\sigma^2} \sim \chi^2_{n-1}} \right.$ $\boxed{\bar{X}, S^2 \text{ are independent}}$

$T_n = \dfrac{Z}{\sqrt{\chi^2_n/n}}$, $Z \sim N(0,1)$

Z and χ^2_n are independent

$T_n \sim Z$ (approximately when n is large)

$$\boxed{\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t_{n-1}}$$

The F- distribution.

If χ^2_n and χ^2_m are independent chi-square random variables with 'n' and 'm' degrees of freedom, then a r.v. $F_{n,m}$ defined by

$$F_{n,m} = \frac{\chi^2_n/n}{\chi^2_m/m}.$$

is said to have an F distribution with n degrees of freedom in the numerator and 'm' degrees of freedom in the denominator.

The pdf. of $F_{n,m}$ is

$$f(x) = \frac{\Gamma\left(\frac{m+n}{2}\right)\left(\frac{n}{m}\right)^{n/2} x^{n/2-1}}{\Gamma\left(\frac{n}{2}\right)\Gamma\left(\frac{m}{2}\right)\left[\frac{n}{m}x+1\right]^{\frac{m+n}{2}}}, \quad 0<x<\infty$$