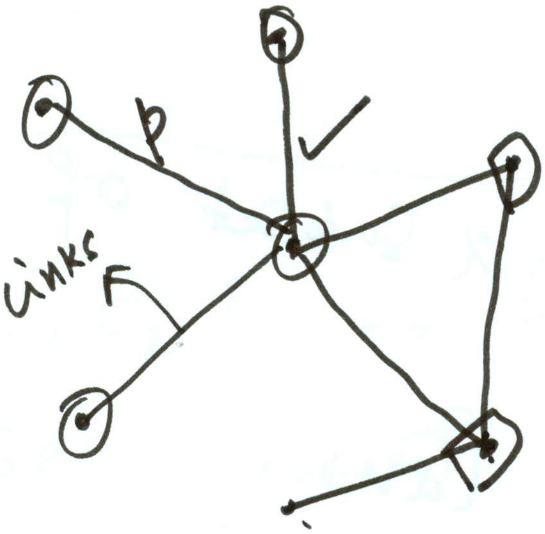
Lec-02

10-1-17

Probability & Statistics.



1959

Paul Erdos

n - nodes

random network

Probabilistic model.

$\Omega \rightarrow$ sample space.

$\hookrightarrow$ mutually exclusive
collectively exhaustive

. Tossing of a coin. $\Omega = \{H, T\}$

Ω : { H, T + $\underline{\text{tep.} < 10°c}$,
T + tep $\geq 10°c$ }

Event = a subset of Ω

<u>Probability law.</u>

is concerned with ~~defi~~ associating a non-negative number with any event!

Axiomatic def.: P : set of events $\rightarrow [0, \infty)$ $[0,1]$

I.) (nonnegative) $P(A) \geq 0$

II.) Let $A \cap B = \emptyset$
then $P(A \cup B) = P(A) + P(B)$

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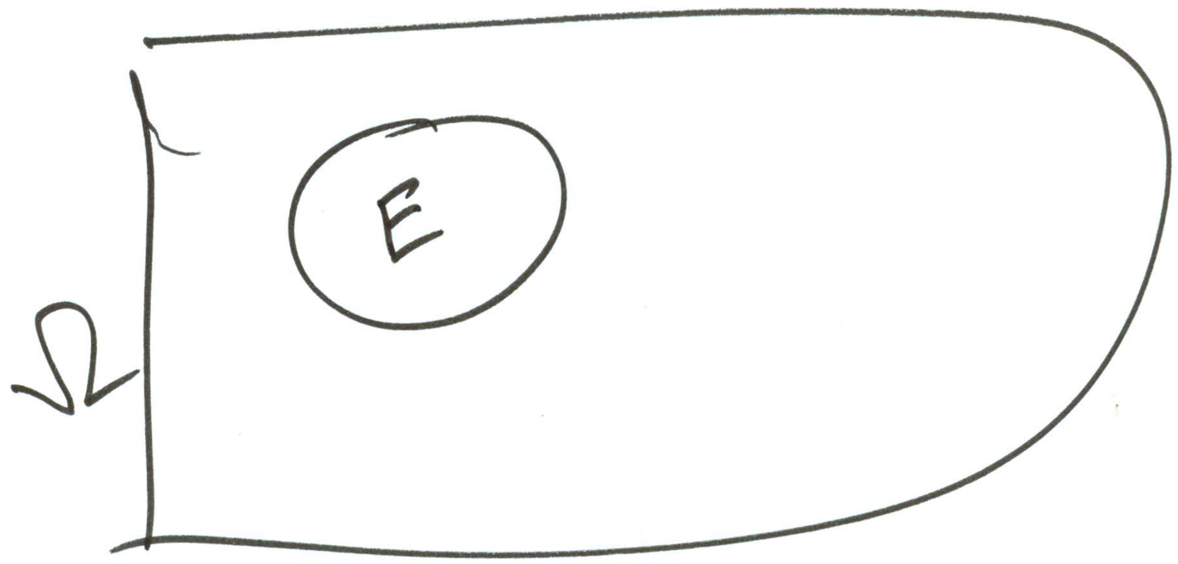


If $E_1, E_2, E_3, \ldots, E_n, \ldots$

is a sequence of mutually disjoint $\left(E_i \cap E_j = \phi,\ i\ \&\ j,\ i \neq j\right)$

$$P\left(\bigcup E_i\right) = \sum P(E_i)$$

III) (Normalization)

$$P(\Omega) = 1.$$

$P : \underbrace{\text{Set of events}} \longrightarrow [0,\infty]$

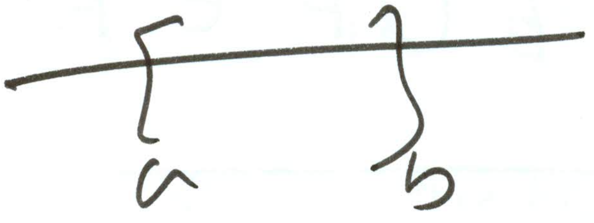
$[0,1]$

$\mathbb{N} \rightarrow$

$f : \{1, 2, \dots, n\} \rightarrow \{1, 2, \dots k\}$

$[a,b] \subseteq \mathbb{R} \longrightarrow \mathbb{R}$

$f(x+h)$ $\qquad \dfrac{f(x+h) - f(x)}{h}$



$E \rightarrow$ random exp.

$\Omega \rightarrow$ sample space.

Algebra. $\mathcal{F} / \mathcal{B}$ — set of events

A field $\mathcal{F}$ of set of subsets of Ω must satisfy:

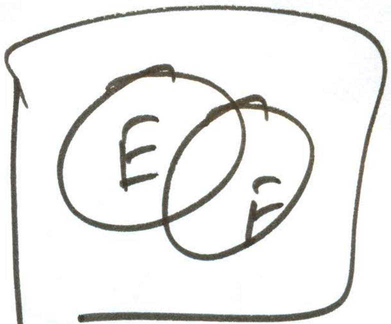
1) if $E \in \mathcal{F}$ then $\underline{E^c = \Omega \setminus E \in \mathcal{F}}$.

II) if $E, F \in \mathcal{F}$ then $E \cup F \in \mathcal{F}$.

<u>Observation:</u>

1) $\underline{\phi \in \mathcal{F} / \cancel{\mathcal{B}}}$ since $\phi = \Omega^c$

2) $E, F \in \mathcal{F}$.



$\Rightarrow E^c, F^c \in \mathcal{F}$.

$\Rightarrow E^c \cup F^c \in \mathcal{F}$

$\Rightarrow (E \cap F)^c \in \mathcal{F}$

$\Rightarrow E \cap F \in \mathcal{F}$

If Ω — is infinite then a set of events is called a Borel field if for any sequence of events $E_1, E_2, \ldots, E_n, \ldots$

1) $\bigcup_i E_i \in \mathcal{F}$.

2) $\bigcap_i E_i \in \mathcal{F}$.

(sigma algebra.)

Prof. Somesh Kumar – coordinator of the course

Probability Space.

triplet – $(\Omega, \mathcal{F}, P)$

sample space

Borel field

Probability law / measure.

$$P : \mathcal{F} \longrightarrow [0, \infty)$$

$\downarrow$ will show! $[0,1]$ (due to properties of P)

1) $P(A) \geq 0$

2) If $A_1, \ldots A_n, \ldots$ is a seqn of mutually exclusive events

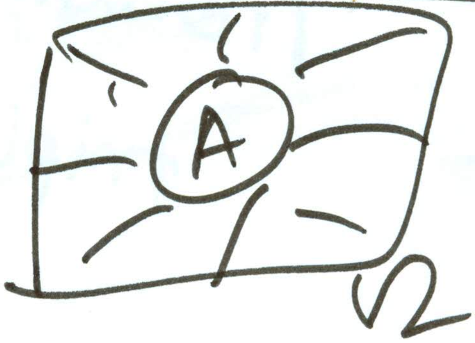
$$P\left(\bigcup_i A_i\right) = \sum_i P(A_i)$$

3) $P(\Omega) = 1$

$P(A_1) + P(A_2) + P(A_3) + \cdots$

Property:

1) $\boxed{P(\phi) = 0}$



Let $A \in \mathcal{F} \Rightarrow A^c \in \mathcal{F}$.

$$A \cup A^c = \Omega$$

$$1 = P(\Omega) = P(A \cup A^c)$$
$$= P(A) + P(A^c).$$

$$\Rightarrow \boxed{P(A^c) = 1 - P(A)}$$

$$\|$$

$$\boxed{P(A) = 1 - \underbrace{P(A^c)}_{\leq 1}}$$

$$\Omega \cup \phi = \Omega$$

$$P(\Omega \cup \phi) = P(\Omega) = 1$$

$$\Rightarrow P(\Omega) + P(\phi) = 1$$

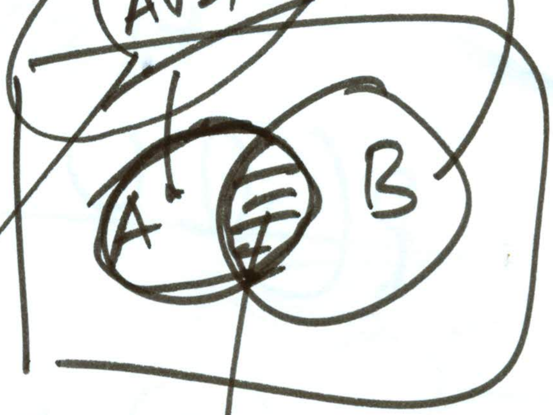
$$\Rightarrow 1 + P(\phi) = 1$$

$$\Rightarrow P(\phi) = 0$$

2) $P(A \cup B) = ?$

$(A \cup B) \setminus B$ $(A \cup B)!$

$= P(A) + P(B) - P(A \cap B)$



Home work! $A \cap B$

$(A \cup B) \setminus B = A \setminus$

Exp. Uniform Discrete model.

Let us toss a coin 3 times.

$$\Omega = \{HHH, HHT, \ldots, TTT\}.$$

Let us assume that all outcomes are equally like.

A = exactly two heads.
$= \{HHT, HTH, THH\}$

$P(A) = ?$

$$= \frac{3}{8} \quad \text{why!}$$

Uniform Discrete Model.

Ω — discrete (finite)

<u>Exp.</u> Tossing a coin

$\Omega = \{H, T\}$

$\mathcal{F} = \{\{H\}, \{T\}, \{H, T\}, \phi\}$

$$\boxed{P(\{H\}) = \frac{1}{2} = P(\{T\})}$$

$P(\Omega) = 1$

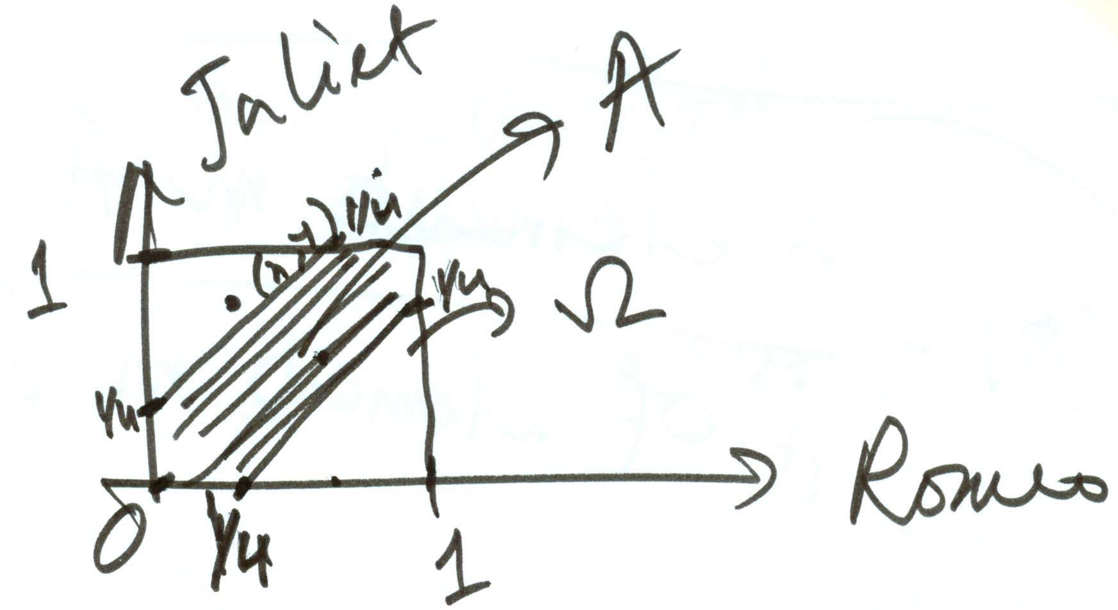
$$P(A) = \frac{\#\text{ elements in } A}{\#\text{ of elements in } \Omega}$$

$$= \frac{3}{8}.$$

<u>Continuous model.</u>

Romeo and Juliet have a date at a given time and each will arrive at the meeting place with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 15 mins. and will leave if the other has not yet arrived. <u>What is the probability that they will meet?</u>

$$P(\downarrow) = \cancel{\frac{1}{16}} \times ?$$



$$P(A) = \frac{7}{16}.$$

Conditional Probability.

$E \to$ random exp.

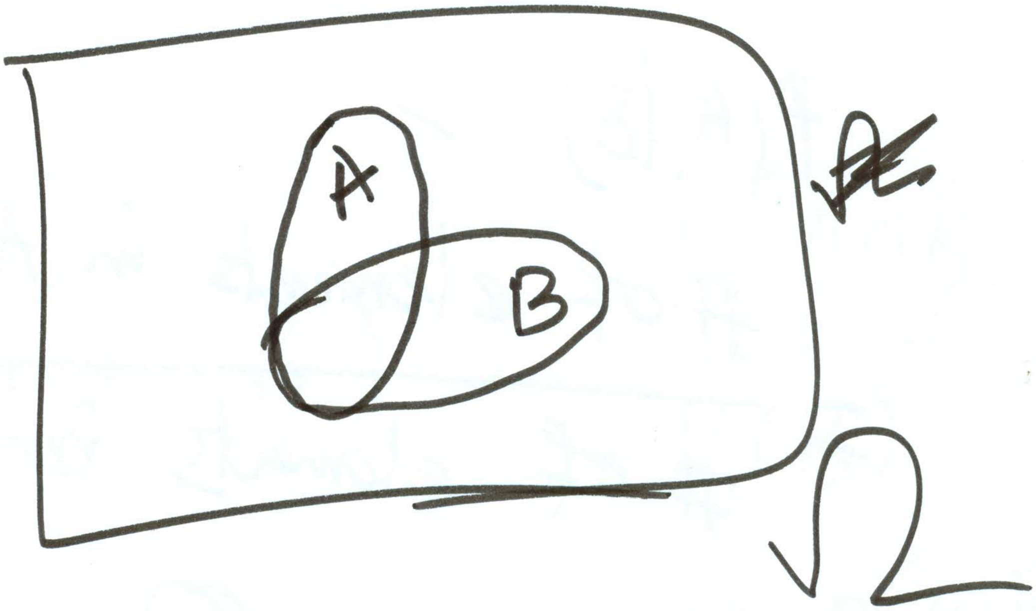
Throwing a die.

~~Tossing a coin~~.

$\boxed{B = \text{the outcome is even}}$

Q. A = the outcome is 6?

$$= P(A) = \frac{1}{3} = P(A|B) = \frac{P(A \cap B)}{P(B)}.$$



~~C~~ Idea: partial information can influence the notion of probability.

Thus we seek to construct a new probability law that takes into account the available knowlede: a probabiliti law that for any event A specifies the condition probability of A given B, denoted by

$$P(A|B)$$

which must satisfy the conditions of below.

$P(A|B)$ -

$$= \frac{\#\text{ of elements in } A\cap B}{\#\text{ of elements in } B}.$$

$$= \frac{\dfrac{\#\text{ of elements in } A\cap B}{|\Omega|}}{\dfrac{\#\text{ of elements in } B}{|\Omega|}}$$

$$= \frac{P(A\cap B)}{P(B)}$$

$$= P(A|B).$$

$\cancel{A(}$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$\Rightarrow P(A\cap B) = P(B)\,P(A|B)$$