

Lecture - 18

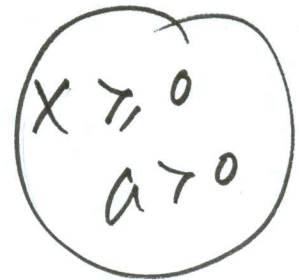
Probability & Statistics.

Moment generating function.
(MGF)

$$\psi(t) = E[e^{tx}]$$

Markov's Inequality.

$$P(X \geq a) \leq \frac{E[X]}{a}$$



Chebyshev's inequality.

$$P(|X - \mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

$$E[X] = \mu$$

$$a > 0$$

$$\text{Var}(X) = \sigma^2$$

Prob. Suppose that it is known that the # of items produced in a factory during a week is a random variable with mean 50.

- (a) What can be said about the probability that this week's production will exceed 75?
- (b) If the variance of a week's production is known to equal 25, then what can be said about the probability that this week's production will be between 40 & 60?

Ans. (a) $\frac{2}{3}$

(b)

$$P \{ |X - 50| \geq 10 \} \leq \frac{\sigma^2}{n^2} = \frac{25}{10^2} = \frac{1}{4}$$

$$\begin{aligned} P \{ |X - 50| \leq 10 \} \\ &= P \{ 40 \leq X \leq 60 \} \\ &\geq 1 - \frac{1}{4} = \frac{3}{4} \end{aligned}$$

Law of Large numbers.

Weak law of large numbers

It states that the probability that the average of the first n terms in a seq. of independent & identically distributed random variable differs by its mean by more than ϵ goes to 0 as $n \rightarrow \infty$.

$$P \left\{ \left| \frac{X_1 + X_2 + \dots + X_n}{n} - \mu \right| > \epsilon \right\} \rightarrow 0 \text{ as } n \rightarrow \infty$$

$$E[X_i] = \mu, \quad \text{Var}(X_i) = \sigma^2$$

