E —

Lecture - 18

Probability & Statistics.

Moment generating function.
(MGF)

$$\psi(t) = E[e^{tX}].$$

Markov's Inequality.

$X \geq 0$
$a > 0$

$$P(X \geq a) \leq \frac{E[X]}{a}$$

Chebyshev's inequality.

$E[X] = \mu$

$$P(|X-\mu| \geq a) \leq \frac{\sigma^2}{a^2}, \quad a > 0.$$

$\mathrm{var}(X) = \sigma^2$

Prob. Suppose that it is known that the # of items produced in a factory during a week is a random variable with mean 50.

(a) What can be said about the probability that this week's production will exceed 75?

(b) If the variance of a week's production is known to equal 25, then what can be said about the probability that this week's production will be between 40 & 60?

Ans. (a) $2/3$ (b)

(5)

$$P\{|X-50| \geq 10\} \leq \frac{\sigma^2}{a^2} = \frac{25}{10^2} = \frac{1}{4}$$

$$P\{|X-50| \leq 10\}$$

$$= P\{40 \leq X \leq 60\}.$$

$$\geq 1 - \frac{1}{4} = \frac{3}{4}.$$

Law of Large numbers.

Weak law of large numbers

It states that the probability that the average of the first n terms in a seq: of independent & identically distributed random variable differs by its mean by more than ε goes to 0 as $n \to \infty$.

$$P\left\{\left|\frac{X_1 + X_2 + \cdots + X_n}{n} - \mu\right| > \varepsilon\right\} \to 0 \text{ as } n \to \infty$$

$$E[X_i] = \mu, \quad \text{var}(X_i) = \sigma^2$$