Lecture - 14

Probability & Statistics.

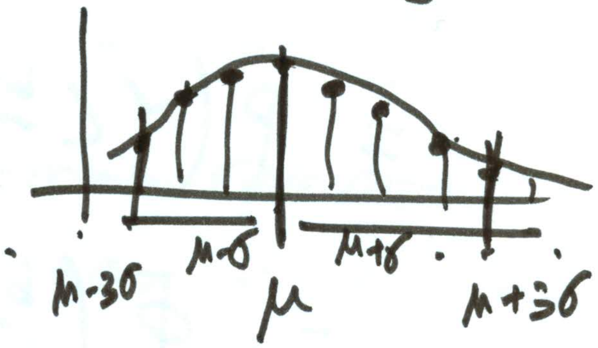
'Normal' (random variable) distribution

Continuous random variable whose pdf

$$P(X=x) = f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty$$

$\sigma > 0$ — $f(x)$ becomes a density fn.

Discovered in 1733 — in order to approximate binomial distribution when n is large.



CDF of $X \sim N(\mu, \sigma^2)$

$$N(0,1) \sim Y = \frac{X-\mu}{\sigma} = -\frac{\mu}{\sigma} + \frac{1}{\sigma} X$$

$$\equiv a + bX$$

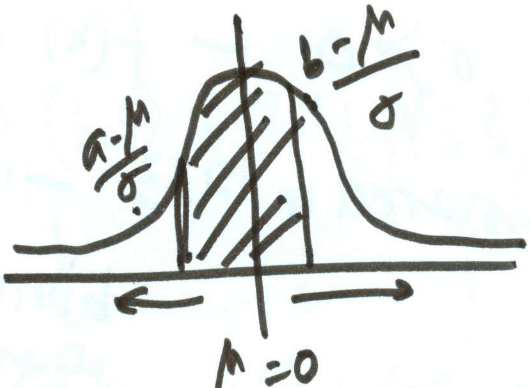
unit normal distn.
standard normal dist.

$E(Y) = 0, \quad var(Y) = 1$

$$P(a < X < b)$$

$$= P\left(\frac{a-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right)$$

$$= P\left(\frac{a-\mu}{\sigma} < Y < \frac{b-\mu}{\sigma}\right)$$

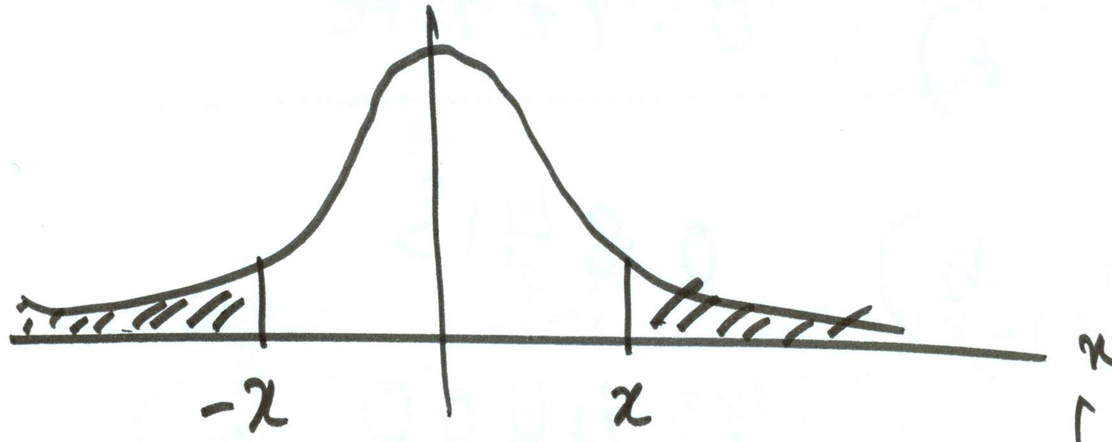


$$= P\left(Y < \frac{b-\mu}{\sigma}\right) - P\left(Y < \frac{a-\mu}{\sigma}\right)$$

$$= \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

where Φ denotes the CDF of $Y \sim N(0,1)$

But Φ is not known still now.



$$\Phi(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt$$

$$\Phi(-x) = P(X \leq -x)$$

$$= 1 - P(X < x)$$

$$= 1 - \Phi(x).$$

<u>Prob</u>. If X is a normal random variable with mean $\mu = 3$ and variance $\sigma^2 = 16$.

$X \sim N(\mu, 16)$

Find

a) $P(X < 11)$

b) $P(X > -1)$

c) $P(2 < X < 7)$