Lecture - 13.

Probability & Statistics.

random variables — discrete / continuous

← Poisson, Exponential, Gamma.

Philosophy - understanding of ~~diff~~ properties of different functions which we call ~~p~~ density functions.

$f(x)$ / $f(k)$

Normal random variable.

A random variable is called normally distributed if its density fn. is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty.$$

μ & σ are the parameters.

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}\, dx$$

$$y = \frac{x-\mu}{\sigma}$$

$$\Rightarrow \quad I = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-y^2/2}\, dy$$

$$I^2 = 1K \int e^{-y^2/2}\, dy \int e^{-z^2/2}\, dz$$

$$= K \iint e^{-\frac{(y^2+z^2)}{2}}\, dy\, dz$$

$y = r\sin\theta, \quad z = r\cos\theta.$

$r\,dr\,d\theta \rightarrow$ H.W.

$$= \frac{1}{2\pi}\int_0^{\infty}\left(\int_0^{2\pi} r e^{-r^2/2}\, d\theta\right) dr$$

$$= \int_0^{\infty} r e^{-\frac{1}{2}r^2}\, dr = 1$$

2) $I = 1$

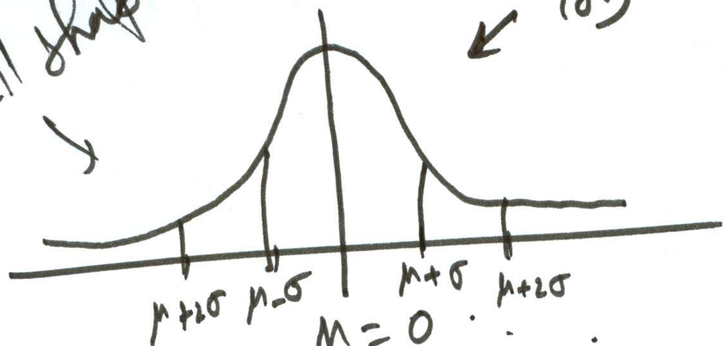
$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2} \quad , -\infty < x < \infty$$

$\mu = 0$ and $\sigma = 1$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2}$$

bell shape

symmetric w.r.t. μ.



In general.

$$\frac{1}{\sqrt{2\pi}\,\sigma} = \frac{0.399}{\sigma}$$