# Lecture - 11

## Probability & Statistics.

### Poisson Distribution.

$$\mathrm{Bin}(n, p)$$

$n \to$ large, $p \to$ small.

Mathematically, we can address situations of this kind by letting $n$ grow while simultaneously decreasing $p$ in a manner that keeps the product '$np$' a constant.

# Poisson Random Variable

$$P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}$$

$k = 0, 1, 2, \ldots\ldots$

Q. Is it a PMF ?

$$\sum_k P(X=k) = 1$$

$\lambda$ can be interpreted as the rate of occurances.

Expectation & variance of $Pois(\lambda)$.

We proved that $E(X) = \lambda$.

$$Var(x) = E[x^2] - (E[x])^2$$

$$E(x^2) = \lambda(1+\lambda)\cancel{e}.$$

$$= \lambda(1+\lambda)\cancel{e} - \lambda^2$$

$$= \lambda.$$

$$e^\lambda = \sum \frac{\lambda^k}{k!}$$

$$\Rightarrow e^\lambda = \sum k\frac{\lambda^{k-1}}{k!}$$

$$\Rightarrow \lambda e^\lambda = \sum \frac{k\lambda^k}{k!}$$

$$\Rightarrow e^\lambda + \lambda e^\lambda = \sum k^2 \frac{\lambda^{k-1}}{k!}$$

$$\Rightarrow \lambda e^\lambda(1+\lambda) = \boxed{\sum k^2 \frac{\lambda^k}{k!}}$$