Probability & Statistics L9

random vectors: $\vec{X} = (X_1, X_2, \ldots, X_n)$

$X_j : \Omega \to \mathbb{R}$ given

$\vec{X} : \Omega \to \mathbb{R}^n$

$\vec{X}(w) = (X_1(w), \ldots, X_n(w))$

$n=2$ $\quad \vec{X} = (X, Y)$

marginal probability distribution $\not\Rightarrow$ joint prob. dist.

but joint prob. dist. $\Rightarrow$ the prob. dist. of the individual random variables.

$$P(X = x_i, Y = y_j) = P(X = x_i)\, P(Y = y_j)$$

$f : \mathbb{R} \to \mathbb{R}$

$f(X)(w) = f(X(w))$

Algebra of Random Variables.

Given $X, Y : \Omega \to \mathbb{R}$

Then
$X+Y : \Omega \to \mathbb{R}$
$(X+Y)(w) = X(w) + Y(w)$.

$X.Y : \Omega \to \mathbb{R}$
$(X.Y)(w) = X(w)\, Y(w)$

$X/Y : \Omega \to \mathbb{R}$
$(X/Y)(w) = \frac{X(w)}{Y(w)}$, $Y(w) \neq 0$

Obs. $X+Y$, $X.Y$, X/Y are all RVs.

Q. What is the prob. dist. of $X+Y$?
=
— do the prob. dist. of X, Y determine the prob. dist. of $X+Y$?

Ans. NO

Recall the exp.

four coins, labelled as $\boxed{\begin{matrix}0&0\\0&0\end{matrix}}$.

each of 0 & 1.

X: without replacement
Y: replacement

X/Y	0	1	
0	1/6	1/3	1/2
1	1/3	1/6	1/2
	1/2	1/2	

X': with replacement
Y': replacement

X'/Y'	0	1	
0	1/4	1/4	1/2
1	1/4	1/4	1/2
	1/2	1/2	

$P_X \equiv P_{X'}$, $P_Y \equiv P_{Y'}$

Q. $P_{X+Y} \stackrel{?}{=} P_{X'+Y'}$??

$P(X+Y=0) = 1/6$
$P(X+Y=1) = 2/3$
$P(X+Y=2) = 1/6$

$P(X'+Y'=0) = 1/4$
$P(X'+Y'=1) = 1/2$
$P(X'+Y'=2) = 1/4$

$P_{X+Y} \neq P_{X'+Y'}$