Probability & Statistics

L6

Conditional Probability measure:

to analyze how occurance of an event influences the occurances of other events!

Recall, $A =$ "sum of two outcomes is 5"

$B =$ the first outcome is even"

$$\boxed{P(A|B) = P(A)}$$

Independent Events:

Given two events A, B, we call them independent if

$$P(A|B) = P(A) \Leftrightarrow P(A \cap B) = P(A)P(B)$$

Note: Let A & B are independent.

Q. A & B^c are independent!!

or A^c & B^c are independent!!

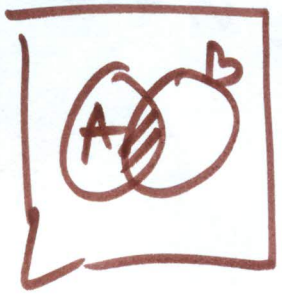
Let us prove that

A & B^c are independent!

$$P(A|B^c) = P(A)$$

$$\Leftrightarrow P(A \cap B^c) = P(A)\,P(B^c).$$

LHS: $P(A)\,P(B^c)$ RHS: $= P(A)\,(1 - P(B))$



$$= P(A) - P(A)P(B)$$

$$= P(A) - P(A \cap B)$$

$$= P(A \mid (A \cap B))$$

$$= P(A \cap B^c)$$

Let $A_1, A_2, \ldots, A_n$ be events.

Pairwise independent:

A_i & A_j for any $i \neq j$, $i, j \in \{1, \ldots, n\}$ are independent.

Independence of events:

For any $I \subseteq \{1, 2, \ldots, n\}$

$$P\left(\bigcap_{j \in I} A_j\right) = \prod_{j \in I} P(A_j)$$