Probability & Statistics

L4

Discrete Probability space.

RE — Ω — countable

$\mathcal{A}$ — $\mathcal{P}(\Omega)$, $\#(\mathcal{A}) = 2^{\#(\Omega)}$

P — PM

$$\Omega = \{\omega_1, \omega_2, \dots\dots\}$$

$$p_i = P(\omega_i)$$

Any such PM,

$$\boxed{p_i \geqslant 0, \quad \sum_i p_i = 1}$$

We are interested to study special PMs which can model many real world random experiments.

Uniform PM

$$\Omega = \{\omega_1, \dots, \omega_N\}$$

$$p_i = P(\{\omega_i\}) = \frac{1}{N}$$

Exp2 A fair coin is labelled with '0' and 1. Toss it n-times.

Q.1 what is the probability that for some fixed $i \leq n$, the ith toss equals '0'?

Ans. $\Omega = \{(\omega_1, \dots, \omega_n) \mid \omega_i \in \{0,1\}\}$

$$\#(\Omega) = 2^n$$

$$A = \{(\omega_1, \dots, \omega_{i-1}, 0, \omega_{i+1}, \dots, \omega_n) \mid \omega_j \in \{0,1\}\}$$

$$\#(A) = 2^{n-1}$$

$$P(A) = \frac{2^{n-1}}{2^n} = \frac{1}{2}.$$

Q2. What is the probability that we observe $k \le n$ times the number '1'?

Ans. $\frac{\binom{n}{k}}{2^n}$? ~~✓~~

$\frac{\binom{n}{k}}{2^{2n}}$ XX

Binomial Distribution.

$\Omega = \{0, 1, \ldots, n\}$.

$0 \le p \le 1$

$p_k = P(\{k\})$, $k \in \Omega$

$= \binom{n}{k} p^k (1-p)^{n-k}$

Check. $p_k \geq 0$, $\sum_{k=0}^{n} p_k = 1$