

Estimation.

- Estimator / Statistic.
- Properties of estimator.
 - unbiased estimator.
 - asymptotically unbiased estimator.
 - consistency of an estimator.
- MSE.

Methods of estimation.

(1) Method of moment estimator.

Observation $(X_1 = x_1, X_2 = x_2 \dots X_n = x_n)$
 from a population with say, pdf $f_{\theta}(x)$
 we are interested to estimate
 $(\theta_1, \theta_2 \dots \theta_k)$.

- Compute
 - (i) Theoretical moments from pdf.
 - (ii) empirical moment from data.
 - (iii) Construct ~~o~~ k equations if you have k unknown parameters.
 - (iv) Solve for k parameter values.

$E(x) = \mu'_1 = 1st\ order\ raw\ moment.$

~~$E(x - \mu) = 0$~~

$E(x^2) = \mu'_2 \quad \text{or} \quad Var(x) = E(x - \mu)^2 = \mu_2$

$$\mu_1' = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\mu_2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\mu_3 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \text{ or } \mu_3' = \frac{1}{n} \sum x_i^3$$

k many equations for k many parameter values.

Gamma (α, λ) .

$$\mu_1' = \frac{\alpha}{\lambda} = \bar{x} \text{ ———— (1)}$$

$$\mu_2 = \frac{\alpha}{\lambda^2} = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

solve for α and λ .

Ex. x_1, \dots, x_n iid $\text{Gamma}(\mu, \sigma^2)$.

We cannot use methods of moment estimation for those cases where theoretical moments does not exist.

eg. $E(|x|)$ $E(x^2)$ as not finite.

