Statistics.

Statistical inference. (Eestimation & Testing of Hypothesis).

Let $(X_1\, X_2 \cdots X_n)$ be random samples from. $f_\theta(\ )$ or $F_\theta(\ )$. where f is the p.d.f. and. F is the c.d.f. of X. Here random sample. $X_1 X_2 \cdots X_n$ are iid. $f_\theta(\ )$. i.e. joint pdf of $(X_1 \cdots X_n)$ at $(x_1, x_2, \cdots, x_n)$. is

$$f(x_1, x_2, \cdots x_n) = \prod_{i=1}^{n} f_\theta(x_i)$$

The family of distributions is defined as.

$$\mathcal{F} = \left\{ f_\theta(\ ) \mid f_\theta(\ ) \text{ is the pdf of } X \text{ and. } \theta \in \Theta \right\}$$

$$\text{or} = \left\{ F_\theta(\ ) \mid F_\theta(\ ) \text{ is the cdf of } X \text{ and } \theta \in \Theta \right\}.$$

e.g.. $\mathcal{B} = \left\{ \binom{n}{x} p^{x} (1-p)^{n-x} \mid n \in \mathbb{N},\ p \in [0,1] \right\}$

parameter space. $\Theta = \mathbb{N} \times [0,1]$

$$\mathcal{N} = \left\{ \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sqrt{2\pi}\,\sigma} \;\middle|\; (\mu, \sigma^2) \in \mathbb{R} \times \mathbb{R}^{+} = \Theta \right\}$$

In parametric estimation problem we want to identify the parameter values of the distribution from given data set.

→ In non-parametric estimation problem we want to identify p.d.f or c.d.f directly from the data without any parametric-setup.

Point estimation in parametric setup.

(I) Dfn of estimation.

(II) Good properties of an estimator.

(III) Methods of estimation. (method of Moment, MLE)

Statistic: A statistic is a function of data but it is free from any unknown parameter.

Let $X_1 \dots X_n$ be random sample from $N(\mu, \sigma^2)$.

(1) μ known: $\sum (x_i - \mu)^2$ is a statistic.

(2). μ unknow: (a) $\sum (x_i - \mu)^2$ is not a statistic.

(b) $\sum (x_i - \bar{X})^2$ is a statistic.

(3) μ, σ^2 unknow. $\dfrac{\sum (x_i - \bar{x})^2}{\sigma^2}$ is not a statistic.