Probability & Statistics

L-18

Prob. Let X & Y have the joint pdf

$$p(x,y) = \exp\left(c + 4x + 4y - \frac{x^2}{2} - \frac{y^2}{2} - \frac{x^2y^2}{2}\right)$$

where c is a constant.

Q. Find $p_{X|Y}(x|y)$.

write this in the form. $e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

Ans. $$p_Y(y) = \int_{-\infty}^{\infty} p(x,y)\,dx$$

$$= \exp\left(c + 4y - \frac{y^2}{2}\right)\int_{-\infty}^{\infty} \exp\left(-\frac{(1+y^2)}{2}x^2 + 4x\right)dx$$

$\vdots$

$$= \frac{\sqrt{2\pi}}{\sqrt{1+y^2}} \exp\left(c + 4y - \frac{y^2}{2} + \frac{8}{1+y^2}\right)$$

$$p_{X|Y}(x|y) = \frac{p(x,y)}{p_Y(y)}$$

$\vdots$

$$\sim N\left(\frac{4}{1+y^2}, \frac{1}{1+y^2}\right)$$

Observation: This is an example where conditional densities are normal but the joint pdf ~~are~~ is not bivariate normal

F-distribution & t-distribution.

Recall Gamma distribution.

$$p_X(x;\alpha,\beta) = \frac{\beta(\beta x)^{\alpha-1}}{\Gamma(\alpha)}e^{-\beta x}$$

$$= \frac{\beta^\alpha x^{\alpha-1}e^{-\beta x}}{\Gamma(\alpha)}, \quad x \geq 0$$

my earlier defn.

$$p_X(x) = \frac{1}{\alpha^\beta\Gamma(\beta)}x^{\beta-1}e^{-x/\alpha}$$

$$E(X^k) = \frac{(\alpha+k-1)\dots\alpha}{\beta^k}$$

$$E(X) = \frac{\alpha}{\beta}, \quad E(X^2) = \frac{(\alpha+1)\alpha}{\beta^2}$$

$$\mathrm{Var}(X) = \frac{\alpha}{\beta^2}.$$

Prop. Let $X_1, X_2, \dots, X_n$ be a seq. of independent random variables

$X_1 \sim \Gamma(\alpha_1,\beta)$, $X_2 \sim \Gamma(\alpha_2,\beta)$, $\dots$, $X_n \sim \Gamma(\alpha_n,\beta)$

then $X_1+X_2+\dots+X_n \sim \Gamma(\alpha_1+\alpha_2+\dots+\alpha_n,\beta)$