Probability & Statistics.

L-17

Central limit theorem (CLT)

$\underbrace{X_1, X_2, \ldots}_{\text{iid}}$ RVs with mean μ, variance σ^2

$E(X^2) < \infty$

$S_n = X_1 + X_2 + \cdots + X_n$

$$P\left(\frac{S_n - n\mu}{\sigma\sqrt{n}} \le x\right) \approx \Phi(x), \quad -\infty \le x < \infty$$

$$\Rightarrow \quad P(S_n \le x) \approx \Phi\left(\frac{x - n\mu}{\sigma\sqrt{n}}\right)$$

$$= \Phi\left(\frac{x - E(S_n)}{\sqrt{Var(X_n)}}\right)$$

very very useful.

normal approximation formula

Bivariate Normal Distribution.

Q. Do ~~not~~ one-dimensional normal distribution and the one-dimensional CLT allow for a generalization to dimension two or higher?

Ans Yes.

As the 1-d normal density is completely determined by its expected value and variance, the bivariate normal density is completely specified by the expected values and the variances of its marginal densities and correlation coefficient.

Bivariate Standard normal.

A random vector (X, Y) is said to have a standard bivariate normal distribution with parameter ρ if it has a joint probability density fn. of the form

$$f(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}}\, e^{-\frac{1}{2}(x^2-2\rho xy+y^2)/(1-\rho^2)}$$

$$-\infty < x, y < \infty$$

$$\rho = \rho(X, Y)$$

Note that $\quad = \text{the correlation coeff.}$

$$f(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{1}{2}x^2} e^{-\frac{1}{2}(y-\rho x)^2/1-\rho^2}$$

Now for any fixed x,

$$g(y) = \frac{1}{\sqrt{1-\rho^2}\sqrt{2\pi}} e^{-\frac{1}{2}(y-\rho x)^2/1-\rho^2}$$

$$\approx N(\rho x, 1-\rho^2)$$

$$\Rightarrow \int_{-\infty}^{\infty} g(y)\, dy = 1.$$

...erefore,

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}, \quad -\infty < x < \infty$$

$$\approx N(0,1).$$

...ing the symmetry in $f(x,y)$, $f_Y(y) \approx N(0,1)$.