Probability & Statistics.

L-14.

$$\vec{X} = (X_1, X_2, \ldots, X_n)$$

$$X_i : \Omega \to \mathbb{R} / \mathbb{R}^n$$

$$\vec{X} : \Omega \to \mathbb{R}^n$$

$$\vec{X}(\omega) = (X_1(\omega), \ldots, X_n(\omega))$$

$$P_{\vec{X}}(B) = P(\vec{X} \in B), \quad B \in \mathcal{B}(\mathbb{R}^n)$$

$\boxed{\text{Joint distribution}}$

Joint density for $\vec{X}$

$$\int \ldots \iint p(x_1, \ldots, x_n)\, dx_n \ldots dx_1 = \underline{1}$$

$$p_{X_j}(x_j) = \iint \cdots \int p(x_1, \ldots x_{j-1}, x_j, \ldots, x_n)\, dx_n \ldots dx_{j+1}\, dx_{j-1} \ldots dx_1$$

marginal density function of X_j.

Ex. Choose by random a point $x = (x_1, x_2, x_3)$ in the unit ball of $\mathbb{R}^3$.

How are the coordinates x_1, x_2 & x_3 are distributed?

$$\vec{X} = (X_1, X_2, X_3)$$

Ans.
$$p_{\vec{X}}(x_1, x_2, x_3) = \begin{cases} \frac{3}{4\pi}, & (x_1, x_2, x_3) \in K \\ 0, & \text{otherwise}. \end{cases}$$

$$p_{X_1}(x_1) = \iint_{x_2^2 + x_3^2 \le 1 - x_1^2} \frac{3}{4\pi} \, dx_2 \, dx_3$$

$$= \frac{3}{4\pi} \cdot \pi (1 - x_1^2)$$

$$= \frac{3}{4}(1 - x_1^2).$$

Independence of continuous random variables.

For the r.v.s. $X_1, \dots X_n$ with densities $p_1, \dots, p_n$, we define a fn. $p: \mathbb{R}^n \to \mathbb{R}$ by $p_{\vec{X}}(x_1, \dots x_n) = p_{X_1}(x_1) \cdots p_{X_n}(x_n)$ then X_i's are independent. $\vec{X} = (X_1, \dots X_n)$