Probability & Statistics.

$X \to$ continuous RV

pdf : $f(x) \geq 0$, $\int_{-\infty}^{\infty} f(x)dx = 1$

PDF : Probability distribution fn.

$F_X(t) = P(X \leq t), t \in \mathbb{R}$

$E(X)$, $V(X)$, moments "μ_n"

Moment generating function (mgf)

$g(t) = E(e^{tX}) < \infty$.

$= \int_{-\infty}^{\infty} e^{tx} f_X(x)\, dx$

$< \infty$.

$$\mu_n = g^{(n)}(0)$$

$\mu_n = E(X^n)$

$= \int_{-\infty}^{\infty} x^n f_X(x)\, dx$

Ex. Let $X \sim U[0,1]$

$$g(t) = \int_0^1 e^{tx}\, dx$$

$$= \frac{e^t - 1}{t}$$

Ex. $X \sim E_\lambda$, $p_X(x) = \lambda e^{-\lambda x}, 0 < x < \infty$

$$g(t) = \int_0^\infty e^{tx} \cdot \lambda e^{-\lambda x}\, dx$$

$$= \lambda \int_0^\infty e^{(t-\lambda)x}\, dx$$

$$= \frac{\lambda}{\lambda - t}$$

Ex. $X \sim N(0,1)$

$$g(t) = \frac{1}{\sqrt{2\pi}} \int e^{tx} \cdot e^{-x^2/2}\, dx$$

$$= e^{t^2/2}$$

[Propo]sition: If X is a bdd continuous [ra]ndom variable then the mgf $g(t)$ [d]etermines the density p_X uniquely.

Functions of continuous RV.

$$Y = aX + b$$

Q. What about the distribution fn. of Y i.e. $F_Y(t)$ given $F_X(t)$!!

$$F_X(t) = P(X \leq t)$$

Case I $a > 0$

$$F_Y(t) = P(Y \leq t)$$
$$= P(aX + b \leq t)$$
$$= P\left(X \leq \frac{t-b}{a}\right)$$
$$= F_X\left(\frac{t-b}{a}\right)$$

Case II $a < 0$

$$F_Y(t) = P(Y \leq t)$$
$$= P\left(X \geq \frac{t-b}{a}\right)$$
$$= 1 - P\left(X < \frac{t-b}{a}\right)$$

$$P\left(X = \frac{t-b}{a}\right) = 0$$

Assuming the continuity.

$$F_Y(t) = 1 - P\left(X \leq \frac{t-b}{a}\right)$$
$$= 1 - F_X\left(\frac{t-b}{a}\right)$$