Probability & Statistics.

L-11

Continuous Prob. Distribution.

$\Omega = \mathbb{R}$

$\mathcal{A}$ = Borel field

P – prob. measures.

Examples of continuous probability measures.

1. Uniform distribution on an interval

Let $I = [\alpha, \beta]$ be a finite interval of real numbers.

Define, $\quad p : \mathbb{R} \to \mathbb{R}$ s.t.

$$p(x) = \begin{cases} \dfrac{1}{\beta - \alpha}, & \text{if } x \in I \\ 0, & \text{otherwise.} \end{cases}$$

Then

$$\int_{-\infty}^{\infty} p(x)\,dx = \int_{\alpha}^{\beta} \frac{1}{\beta-\alpha}\,dx = 1$$

$$P([a,b]) = P(a \le X \le b)$$

$\boxed{p_X(x) = p(x).}$

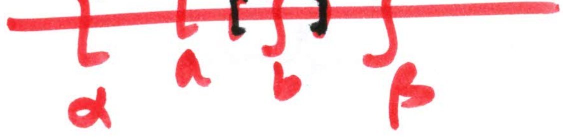
$$= \int_a^b p_X(x)\,dx.$$

$\boxed{= P(X = x)}$

$$= \int_a^b \frac{1}{\beta-\alpha}\,dx$$

$\boxed{p(X = x_j) = p_j \text{ for discrete}}$

$$= \frac{b-a}{\beta-\alpha}$$



$$= \frac{\text{length of } [a,b]}{\text{Length of } [\alpha,\beta]}$$

This explains why P is uniform distribution.
The prob. of an interval $[a,b] \subseteq I$ depends only on its length, not on its position.