Probability & Statistics.

L-10

X, I don't know the P_X
but $\underbrace{E(X) \text{ \& } V(X)}_{\text{do they determine the } P_X \text{ uniquely.}}$ are known.

$$P_X = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1/4 & 1/2 & 0 & 0 & 1/4 \end{pmatrix}$$

$$P_Y = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1/4 & 0 & 0 & 1/2 & 1/4 & 0 \end{pmatrix}$$

$$E(X) = \frac{7}{2} = E(Y), \quad V(X) = \frac{9}{4} = V(Y)$$

Momoment Generating Function (MGF).

$\mu_n = E(X^n) \rightarrow$ nth moment of X.

$$g(t) = E(e^{tX}) = \sum_{k=0}^{\infty} \mu_k \frac{t^k}{k!}$$

$$= \sum_{j=0}^{\infty} e^{t x_j} p(X = x_j)$$

Then $\frac{d^n}{dt^n} g(t)\Big|_{t=0} = g^{(n)}(0)$

$$= \sum_{K=n}^{\infty} \frac{K! \; \mu_K \, t^{K-n}}{(K-n)! \; K!}\Bigg|_{t=0}$$

$$= \mu_n .$$

Exp. $B_{n,p}$.

$$g(t) = \sum_{j=0}^{n} e^{tj} \binom{n}{j} p^j (1-p)^{n-j}$$

$$= \sum_{j=0}^{n} \binom{n}{j} (pe^t)^j (1-p)^{n-j}$$

$$= \left[pe^t + (1-p)\right]^n$$

exp. G_p

$$g(t) = \sum_{j=1}^{\infty} e^{tj} p(1-p)^{j-1}$$

$$= \frac{pe^t}{1-(1-p)e^t}$$

Exp. $\underline{Pois_\lambda}$

$$g(t) = \sum_{j=0}^{\infty} e^{tj} \frac{e^{-\lambda}\lambda^j}{j!}$$

$$= e^{\lambda(e^t-1)}$$

Remark. Using MGF for discrete Random variables with finite outcomes, its distribution function can be uniquely determined.