Probability & Statistics . L1

Sec-1 (AG+IM+MI)

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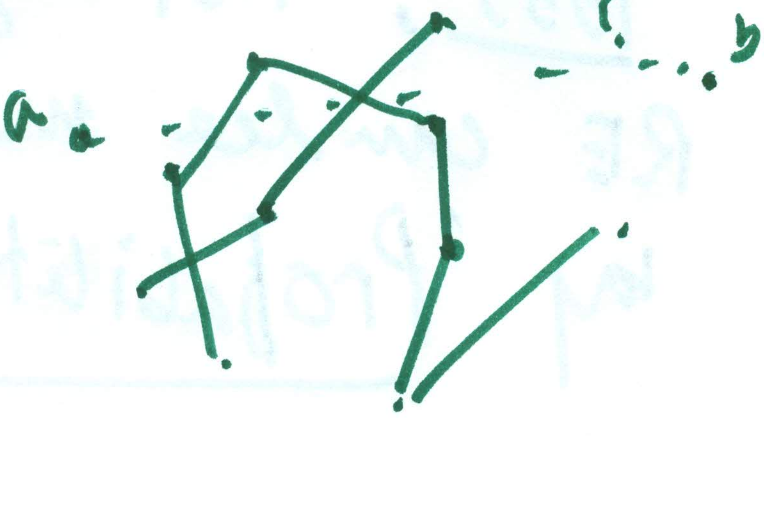
Midsem -	30
End Sem -	50
Class test -	20

Prof. Swanand R Khare



The theory of probability is concerned with ~~exp~~ modelling experiments which have non-deterministic results/outcomes. = 'random'

'random experiments'.

1933, Kolmogorov,
RE can be modelled by 'Probability Spaces'

$$(\Omega, \mathcal{A}, P)$$

sample space

$P: \mathcal{A} \to [0,1)$ probability measure.

σ-field of events

$\Omega \equiv$ the non-empty set of all possible outcomes. (but Not so always)

<u>Exp.</u> <u>Rolling a Die</u>.

$$\Omega = \{1, 2, 3, \ldots, 6\}$$

<u>Exp.</u> <u>Rolling a Die untill '6' appears</u>

$$\Omega = \{1, 2, 3, \ldots \ldots\}$$

Exp 3. A light blub is switched on at time zero and burns for a certain period of time.
At some 'random' time t, it burns out.

$$\Omega = (0, \infty) \text{ or}$$
$$[0, \infty) \text{ if blub is } \underline{\text{defective}}.$$

Event: Subsets of the sample space.

Exp. $\Omega = \{1, 2, \ldots, 6\}$.

Q. How many events are there?

$\{1, 2\}$, $\{2, 3\}$, $\{1, 2, 3\}$,

$2^6 - 1$ events are possible.

$\Omega = \{1, \ldots, 6\}$.

$\{1\}, \{2\}, \ldots \{6\}$

$\downarrow$ elementary events.

<u>Remark</u>. The 'event' A occurs.

any $\omega \in A$ occurs.

if $\omega \notin A$, ω does not occur.

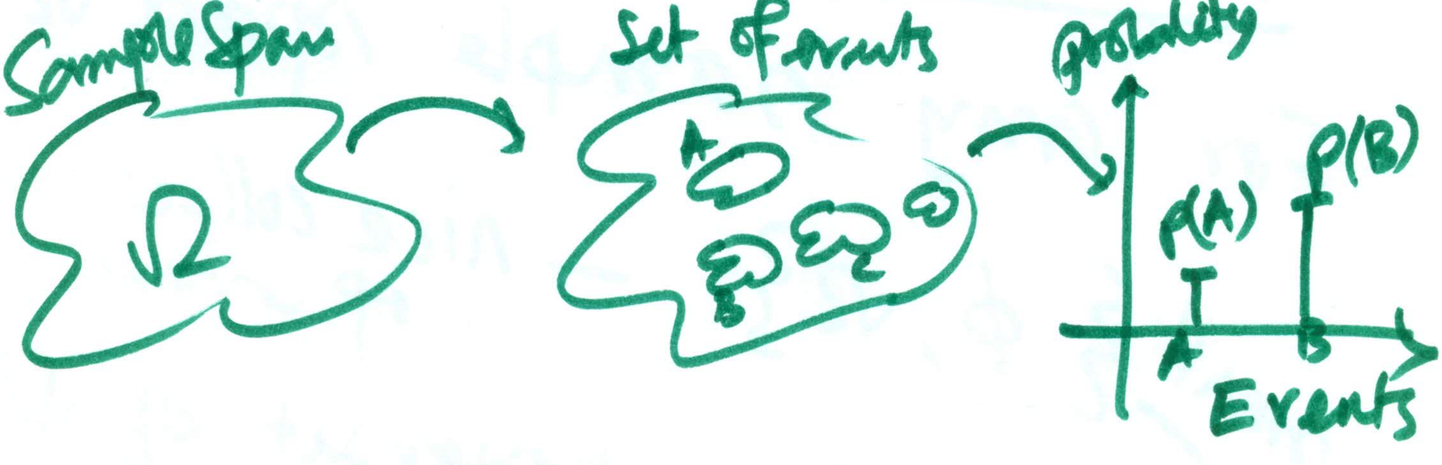
For any sample space Ω,

too small $\{\phi, \Omega\}$ — nice collection of events.

too big ✓ $P(\Omega)$ — powerset of Ω, is a nice collection of events.

Both the collections of events are not enough to analize a particular Random Experiment.

Q. What is a collection of events that can be considered as a 'good' choice of events for a Random experiment?



σ-field of events.

A collection $\mathcal{A}$ of subsets of Ω is called a σ-field if

(i) $\phi \in \mathcal{A}$

(ii) if $A \in \mathcal{A}$ then $A^c \in \mathcal{A}$.

(iii) for countably many $A_1, A_2, \ldots$ in $\mathcal{A}$,

$$\bigcup_{j=1}^{\infty} A_j \in \mathcal{A}.$$

$\Omega = \{1, 2, \ldots, 6\}$

$\mathcal{A} = \{\{1,3,5\}, \{2,4,6\}, \phi, \Omega\}$

$\mathcal{A} = \{\{1,2\}, \{3,4,5,6\}, \phi, \Omega\}$

?? $\mathcal{A} = \{\{1,2\}, \{1,3\}, \{3,4,5,6\}, \{2,4,5,6\}, \phi, \Omega\}$

$$\alpha \acute{U} + \beta \acute{V} \in V$$

$\swarrow \qquad L$

<u>Obs.</u> Let $\mathcal{A}$ be a σ-field of a random experiment

(i) $\Omega \in \mathcal{A}$

(ii) if $A_1, A_2, \dots, A_n \in \mathcal{A}$ then $\bigcup_{j=1}^{n} A_j \in \mathcal{A}$.

Pf. $A_{n+1} = \phi$, $A_{n+2} = \phi$,

$A_k = \Phi$, $k \geq n+1$

$$\bigcup_{j=1}^{n} A_j = \bigcup_{j=1}^{\infty} A_j \in \mathcal{A}$$

(iii) if $A_1, A_2, \dots \in \mathcal{A}$ then $\bigcap_{j=1}^{\infty} A_j \in \mathcal{A}$.

Pf. H.W.

subsets of Ω . . .

Given a random experiment and a collection of events, say $\mathcal{C} \subseteq \mathcal{P}(\Omega)$

$$\mathcal{A}_{\bar{\Phi}} = \bigcap_{\mathcal{A}' \in \bar{\Phi}} \mathcal{A}'$$

where $\bar{\Phi} = \{ \mathcal{A}' \subseteq \mathcal{P}(\Omega) \mid \mathcal{C} \subseteq \mathcal{A}', \ \mathcal{A}' \text{ is a } \sigma\text{-field} \}$