**Probability and Statistics**
**Hints/Solutions to Assignment No. 3**

1. $P(\text{Ruby wins}) = \dfrac{\binom{4}{2}\binom{3}{1} + \binom{4}{3}}{\binom{7}{3}} = \dfrac{22}{35}.$

2. Suppose n missiles are fired and X is the number of successful hits. Then X ~ Bin(n, 0.75). We want n such that $P(X \geq 3) \geq 0.95$, or $P(X = 0) + P(X = 1) + P(X = 2) \leq 0.05$. This is equivalent to $10(9n^2 - 3n + 2) \leq 4^n$. The smallest value of n for which this is satisfied is n =6.
3. Let $P_n$ denote the probability of an n-component system to operate effectively. Then $P_3 = 1-(1-p)^3$ *and* $P_6 = 1-(1-p)^6 - \binom{6}{1} p(1-p)^5$. Now $P_6 - P_3 \geq 0$, if

   $\dfrac{9-\sqrt{21}}{10} \leq p \leq 1.$

4. 
$$P(\textit{returning a packet}) = P(X > 1) = 1 - P(X = 0) - P(X = 1)$$
$$= 1 - (0.99)^{10} - 10(0.99)^9(0.01),\ \textit{say}.$$
   Let Y denote the number of packets returned. Then Y ~ Bin(3, p).
   $P(Y = 0) + P(Y = 1) = 0.9999455.$
5. 
$$P(Y = 2000\,j) = \frac{e^{-2}(2)^j}{j!}, \qquad j = 0, 1, \cdots, 9,$$
$$= \sum_{j=10}^{\infty} \frac{e^{-2}(2)^j}{j!}, \quad j = 10.$$
6. Let A be the event that person gets a cold and B denote the event that the drug is beneficial to him. Let X be the number of times an individual contracts the cold in a year. Then $X \mid B \sim P(2)$, and $X \mid B^C \sim P(3)$.
   $P(A^C \mid B) = P(X = 0 \mid B) = e^{-2}$. $P(A^C \mid B^C) = P(X = 0 \mid B^C) = e^{-3}$.
   Using Bayes Theorem $P(B \mid A^C) = 0.89$.

7. Let X be the number of errors. Then X ~ P(300). Ley Y be the number of errors in 2% of the pages. Then Y ~ P(6). The required probability is $P(Y \leq 4) = 0.285$.
8. 5/9.

9. Let X denote the life of a bulb. Then $P(X > 200) = e^{-2}$. Let Y denote the number of bulbs working after 200 hours. Then Y ~ Bin (20, $e^{-2}$). So $P(Y \geq 2) = 0.2254$..
10. $P(X > 6) = P(X > 6 \mid I)P(I) + P(X > 6 \mid II)P(II) = e^{-1}.\dfrac{1}{5} + e^{-3}.\dfrac{4}{5}$
    $= 0.1134.$
11. $P(\text{system fails before time t}) = 1 - \exp\{-t\sum_{i=1}^{n} \lambda_i\}.$

P(only component j fails before time t | system fails before time t)

$$= \frac{(1-\exp\{-\lambda_j t\})\exp\{-t\sum_{\substack{i=1\\ i\neq j}}^{n}\lambda_i\}}{1-\exp\{-t\sum_{i=1}^{n}\lambda_i\}}.$$

12. $r = 4, \lambda = 1/5$, Reqd Prob $= 0.3528$.

13. $P(X \le 100 \mid X \ge 90) = 0.15$ gives $\alpha = 0.0000855$.

$P(X > 80) = \exp\{-0.0000855 \times 80^2\} = 0.5786.$

14. $\mu = 58.13$, $\sigma = 10.26$, Percentage of students getting I[st] Class = 37.86, Percentage of students getting II[nd] Class = 47.14.

16. Let X denote the diameter (in cm) of a ball bearing. Then $X \sim N(3, 0.005^2)$.

$$P(\text{ ball bearing is scrapped}) = 1 - P(2.99 < X < 3.01)$$
$$= 2\Phi(-2) = 0.0455.$$

17. $P(0.895 < X < 0.905) = P(-1.66 < Z < 1.66) = 2\Phi(1.66) - 1 = 0.903.$

So the percentage of defectives = 100 *0.097 = 9.7%.

When $X \sim N(0.9, \sigma^2)$, then $P(0.895 < X < 0.905) \ge 0.99$ is equivalent to

$\Phi(0.005/\sigma) \ge 0.995$, or $\sigma \le 0.00194$.

18. Let X denote the height (in cm.) that univ. high jumper jumps.

Then X ~ N(200, 100). Let c be such that P(X > c) =0.95.

Then (200 – c) / 10 = 1.645 and so c = 183.55 cm.

Further let d be such that P( X > d) = 0.1.

Then (200 – d)/10 = -1. 28 and so d = 212.80 cm.

19. Let X denote the marks. Then X ~ N(74, 62.41).

Ans. (a) 64 (b) 86 (c) 77

20.

$$E(\text{Profit}) = C_0 P(6 \le X \le 8) - C_1 P(X < 6) - C_2 P(X > 8)$$
$$= C_0\{\Phi(8-\mu) - \Phi(6-\mu)\} - C_1\Phi(6-\mu) - C_2\Phi(\mu - 8).$$

Using derivatives and simplifying we obtain the maximizing choice of $\mu$ as

$$\mu^* = 7 + \frac{1}{2}\ln\left(\frac{C_1 + C_0}{C_2 + C_0}\right).$$

21. $\ln Y \sim N(0.8, 0.01)$. So $P(Y > 2.7) = P(\ln Y > 0.9933) = P(Z > 1.93) = 0.0268.$

Let c be such that $P(0.8 - c < \ln Y < 0.8 + c) = 0.95$. This is equivalent to

$P(-\frac{c}{0.1} < Z < \frac{c}{0.1}) = 0.95$, or $\Phi\left(\frac{c}{0.1}\right) = 0.975$, so $c = 0.196$. Therefore

$P(1.8294 < Y < 2.7074) = 0.95.$