Probability and Statistics
Assignment No. 3

1. Ruby and Mini tied for the first place in a beauty contest. The winner is to be decided by the majority opinion of a panel of three judges chosen at random from a group of seven judges. If four of these judges favour Ruby and three favour Mini, what is the probability that Ruby will be declared the winner.
2. In a precision bombing attack there is a **50%** chance that a bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give at least **99%** chance of completely destroying the target?
3. A communication system consists of n components each of which function independently with probability p. The entire system will be able to operate effectively, if at least one-half of its components function. For what values of p, a 5-component system more likely to operate effectively than a 3-component system?
4. The diskettes produced by a certain company are defective with probability 0.01, independently of each other. The company sells the diskettes in packs of size 10 and offers a money-back guarantee if more than one of the 10 diskettes in the pack is found to be defective. If you buy 3 packs, what is the probability that at most one pack will be returned.
5. The number X of computers that a hardware store sells in a week obeys the Poisson distribution with $\lambda = 2$. The profit on each computer is Rs. 2000.00. If at the time of opening the store 10 computers are in stock (with no replenishment during the daytime), the profit from the sale of the computers during the day is $Y = 2000 \min(X, 10)$. Find the probability distribution of Y.
6. The number of times that an individual contracts cold in a given year is a Poisson random variable with parameter $\lambda = 3$. Suppose that a new drug has been just marketed that reduces the parameter λ to 2 for 75% of the population. For the other 25% of the population the drug has no appreciable effect on the cold. If an individual tries the drug for a year and has no cold in that time, how likely is it that the drug is beneficial for him?
7. A large microprocessor chip contains multiple copies of circuits. If a circuit fails, the chip knows it and knows how to select the proper logic to repair itself. The average number of defects per chip is 300. What is the probability that no more than 4 defects will be found in a randomly selected area that comprises 2% of the total surface area?
8. A boy and a girl decide to meet between 5 and 6 p.m. in a park. They decide not to wait for the other for more than 20 minutes. Assuming arrivals to be independent and uniformly distributed, find the probability that they will meet.
9. A contractor has found through experience that the low bid for a job is a uniform random variable on $(\frac{3}{4}C, 2C)$, where C is the contractor’s cost estimate (no profit, no loss) of the job. The profit is defined as zero if the contractor does not get the job and as the difference between his bid and his cost estimate C if he gets the job. What should he bid (in terms of C) in order to maximize his expected profit?
10. A small industrial unit has **10** bulbs whose lifetimes are independent exponentially distributed with mean **50** hours. If all the bulbs are used at a time, find the probability that even after **100** hours there are at least two bulbs working.

11. The time to failure in months, X, of the light bulbs produced at two manufacturing plants A and B obeys exponential distribution with means 5 and 2 months respectively. Plant B produces three times as many bulbs as plant A. The bulbs indistinguishable to eye are intermingled and sold. What is the probability that a bulb purchased at random will burn at least 5 months?
12. A series system has n independent components. For i = 1,…, n, the lifetime X_i of the i^{th} component is exponentially distributed with parameter λ_i. If the system has failed before time t what is the probability the failure was caused only by component j (j = 1,…,n).
13. The lead time for orders of diodes from a certain manufacturer is known to have a gamma distribution with a mean of 20 days and a standard deviation of 10 days. Determine the probability of receiving an order within 15 days of placement date.
14. The lifetime X in hours of a component is modelled as a Weibull distribution with $\beta = 2$. Starting with a large number of components it is observed that 15% of the components that have lasted 90 hours fail before 100 hours. Determine the parameter α. Further determine the probability that a component is working after 80 hours.
15. In an examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less than 45%, between 45% and 60%, between 60% and 75% and above 75% respectively. In a particular year 10% of students failed in the examination and 5% students got distinction. Assuming the marks to be normally distributed, find the percentage of students who get first class and second class respectively.
16. In an industrial process the diameter of a ball bearing is an important component. The buyer sets specifications on the diameter to be 3.0 ± 0.01 cm. The diameter has a normal distribution with mean 3 cm. and s.d. 0.005 cm. On the average how many manufactured balls will be scrapped?
17. The width of a duralumin forging is (in inches) normally distributed with $\mu = 0.9$ and $\sigma = 0.003$. The specification limits were given as 0.9 ± 0.005. What percentage of forgings will be defective? What is the maximum allowable value of σ that will permit no more than 1 defective in 100 when the widths are $N(0.9, \sigma^2)$?
18. The height a university high jumper will clear, each time he jumps, is a normal r.v. with mean 2 meters and s.d. 10 cm. What is the greatest height that he will jump with probability 0.95? What is the height that he will clear only 10% of the time?
19. If a set of marks on a Statistics exam is approximately N(74, 62.41), find
    a) the lowest passing grade if the lowest 10% of the students are given F’s;
    b) the highest B if the top 5% of the students are given A’s;
    c) the lowest B if the top 10% of the students are given A’s and the next 25% are given B’s.
20. The diameters X of a ball-bearing are distributed normally with mean μ and standard deviation unity, If X lies in the specification limits of 6 to 8 inches, a profit of Rs. C_0 is gained. However, in case $X < 6$ or $X > 8$, there is a loss of Rs. C_1 or Rs. C_2 respectively. Find the value of μ that maximizes the expected profit.
21. Let Y denote the diameter in mm. of certain type of nuts. Assume that Y has a log-normal distribution with parameters $\mu = 0.8$ and $\sigma = 0.1$. Find the probability that a randomly selected nut has diameter more than 2.7 mm. Between what two values will Y fall with probability 0.95?