Probability and Statistics
Assignment No. 2

1. Check if the following functions define cdf's :
   (a) $F(x) = 0$, if $x < 0$, $= x$, if $0 \le x \le 1/2$, and $= 1$, if $x > 1/2$.
   (b) $F(x) = (1/\pi) \tan^{-1}x$, $-\infty < x < \infty$.
   (c) $F(x) = 0$, if $x \le 1$, and $= 1 - (1/x)$, if $x > 1$.
   (d) $F(x) = 1 - e^{-x}$, if $x \ge 0$, and $= 0$, if $x < 0$.
2. Let X be a r.v. with the cdf given by :
   $F(x) = 0$, if $x < 0$;
   $= x/4$, if $0 \le x < 1$,
   $= (x+1)/4$, if $1 \le x < 2$;
   $= 11/12$; if $2 \le x < 3$,
   $= 1$, if $x \ge 3$.
   Find $P(1/2 \le X \le 5/2)$, $P(1 < X < 3)$, E(X), V(X) and median of X.

3. A burnt out bulb was mistakenly placed in a box containing 3 good bulbs. In order to locate the bad bulb, the bulbs are randomly tested one by one, without replacement. Let X denote the number of bulbs tested to determine the bad bulb. Find the probability mass function and the cumulative distribution function of X.

4. An guinea pig either dies(D) or survives(S) in the course of a surgical experiment. The experiment is to be performed first with two guinea pigs. If both survive, no further trials are to be made. If exactly one guinea pig survives, one more guinea pig is to undergo the experiment. If both guinea pigs die, two additional guinea pigs are to be tried. Assuming that the trials are independent and that the probability of survival in each trial is 2/3 find the probability distributions of the number of survivals and the number of deaths.

5. Let X and Y be two independent random variables with respective m.g.f.'s given by $M_X(s) = \frac{(1+e^s)^3}{8}$ and $M_Y(t) = e^{e^t - 1}$. Find $P(X + Y = 1)$.

6. Let X be a continuous r.v. with pdf given by

   $$f(x) = \begin{cases} x/2, & \text{if } 0 \le x \le 1; \\ 1/2, & \text{if } 1 < x \le 2; \\ (3-x)/2 & \text{if } 2 < x \le 3; \\ 0, & \text{elsewhere.} \end{cases}$$

   Find cdf, mean, variance and the median of X.

7. Suppose that a particle is equally likely to release one, two or three other particles and suppose that these second generation particles are, in turn, equally likely to release one, two or three third generation particles. Find the pmf of the number of third generation particles.

8. To determine whether or not they have a certain blood disease, 100 people are to have their blood tested. However, rather than testing each individual separately, it

has been decided first to group the people in batches of 10. The blood samples of the 10 people in each batch will be pooled and analyzed together. If the test is negative, one test will suffice, else each of the ten people will be individually tested. Assuming the incidence of disease to be 1% for the population, find the expected number of tests required for each batch.

9. A certain community is composed of m families, n_i of which have i children, $\sum_{i=1}^{r} n_i = m$. If one of the families is randomly chosen, let X denote the number of children in that family. Find E(X). If one of the $\sum_{i=1}^{r} i\, n_i$ children is randomly chosen, let Y denote the total number of children in the family of that child. Find E(Y). Show that $E(Y) \geq E(X)$.

10. Let X be a discrete random variable with $p_X(1) = \frac{1+3d}{4}$, $p_X(2) = \frac{1-d}{4}$, $p_X(3) = \frac{1+2d}{4}$ and $p_X(4) = \frac{1-4d}{4}$. For what values of d, does $p_X(x)$ describe a valid probability mass function? Further determine the value of d for which Var(X) is a minimum.

11. Let X denote the number of accidents in a factory per week having p.m.f.

$$p_X(x) = \frac{k}{(x+1)(x+2)}, \quad x = 0, 1, 2, ...$$

Find the value of k, cdf of X, E(X) and the median of X.

12. Let the distribution of scores on an IQ test have mean 100 and s.d. 16. Use Chebyshev's inequality to show that the probability of a student having IQ above 148 or below 52 is at most 1/9.