Probability and Statistics
Assignment No. 9

1. Let X ~ Bin (1,p), $0 \leq p \leq ½$. Find the MLE and MME of p and check if they are unbiased.

2. Consider a random sample $X_1,...,X_n$ from a double exponential distribution with density

$$f(x,\theta,\sigma) = \frac{1}{\sigma}\exp\{-\frac{|x-\theta|}{\sigma}\} \ , \ x > \theta, \theta \in R, \sigma > 0.$$

   Find MME's and MLE's of θ and σ. Discuss the unbiasedness of MME's.

3. Let $X_1,...,X_n$ be a random sample from a $N(5,\sigma^2)$ population. Find the MME and the MLE of σ^2. Check their unbiasedness and consistency.

4. Assume the number of new car sales per day by a dealer is a Poisson r.v. with parameter μ. If in 20 days he sold 30 cars, find the MLE of μ.

5. Stress tests are conducted on fibreglass rods used in communication networks. The r.v. X is the distance in inches from the anchored end of the rod to the crack location when the rod is subjected to extreme stress. Let X ~ U (0, b) . The data obtained on 10 test rods are : 10, 7, 11, 12, 8, 8, 9, 10, ,9, 13. (a) Find an unbiased estimate for the average distance from the anchored end of the rod to the crack. (b) Find an unbiased estimate for Var (X). (c) Find an unbiased estimate for b. (d) Find an estimate for s.d. of X. Is this estimate unbiased?

6. An acid solution made by mixing a powder compound with water is used to etch aluminium. The pH of the solution, X, will vary due to slight variations in the amount of water used, the potency of the dry compound, and the pH of the water itself. Assume that X is a gamma r.v. with p and α unknown. On the basis of 15 observations on X given below evaluate the MME's of p, α, E(X) and Var(X) : 1.2, 2.0, 1.6, 1.8, 1.1, 2.5, 2.1, 2.6, 2.2, 1.7, 1.5, 1.7, 2.0, 3.0, 1.8.

7. The lifetimes of a component are assumed to be exponential with parameter λ. Ten of these components were placed on test independently. Due to a technical fault, the only information that was saved was that three components failed before 100 hours. Find the MLE of λ.

8. Find a **90%** confidence interval for the mean of a normal distribution with **σ = 3**, given the sample **(3.3, − 0.3, − 0.6, − 0.9)**. What would be the confidence interval if σ were unknown?

9. If **31** measurements of boiling point of sulphur have a standard deviation **0.83^0** Celsius, construct a **98%** confidence interval for the true standard deviation of such measurements.

10. In pouring glass for use in automobile windshields uniformity of thickness is desirable to prevent distortion. If a random sample of 100 wind shields yields a sample standard deviation of 0.01 inch, construct a 95% confidence interval on the standard deviation in thickness.

11. Five pairs of tests are conducted to compare two methods of making rope. Each sample batch contains enough hemp to make two ropes. The tensile strength measurements are:

Tests	1	2	3	4	5
Method 1	14	12	18	16	15
Method 2	16	15	17	16	14

Find a 95% confidence interval for the mean difference in tensile strengths between ropes made by two methods.

12. Independent random samples of sizes 36 and 49 are taken from two normal populations having known standard deviations of 1.2 and 1.8 respectively. Based on these samples find a 90% confidence interval for the difference in the means of two populations.

13. Assume that there is a constant probability p that a towel manufactured by machine in a factory will be declared a "second" due to some defect. If in 1000 towels selected at random from one day's output, it is found that 30 are "seconds", find a 95% confidence interval for p.