Proability and Statistics

MA20205

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Autumn 2022-23, IIT Kharagpur

Lecture 4
August 23, 2022

Random Variables

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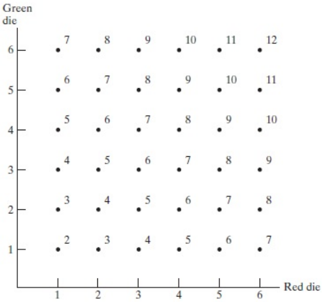
- $\rightarrow$ Discrete : if R_X is countable
- $\rightarrow$ Continuous: if R_X is an interval or union of intervals (uncountable)
- $\rightarrow$ Mixed

Discrete random variables

Example: rolling a pair of six-faced dice, a green and a red one



Let X denote the sum of outcomes in a roll. Then

$$X = 9 \equiv \{(6,3),(4,5),(5,4),(3,6)\}$$
$$2 \leq X < 4 \equiv \{(1,1),(1,2),(2,1)\}$$

Discrete random variables

Then

x	$P(X = x)$
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
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Thus the function $f : \mathbb{R} \to [0, 1]$ given by

$$f(x) = P(X = x) = \begin{cases} \dfrac{6 - |x - 7|}{36} & \text{if } x = 2, 3, \ldots, 12 \\ 0 & \text{otherwise} \end{cases}$$

capture the 'information' about probability of events

Probability mass/density function (pmf/pdf)

Let X be a discrete random variable. Then the function $f : R_X \to \mathbb{R}$ defined by

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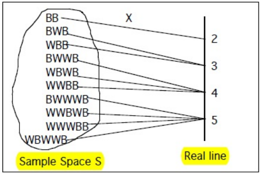

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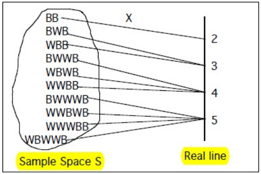

$$f(x) = \frac{x-1}{10}, x = 2, 3, 4, 5$$

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(a) $f(x) \geq 0$ for any $x \in R_X$

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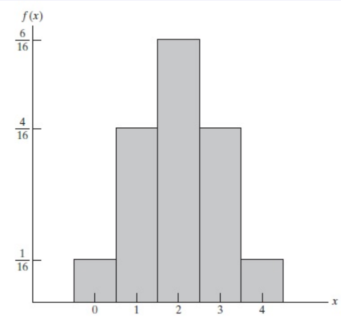
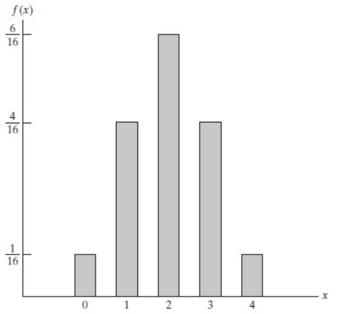
Problems..

Discrete random variables

pdf diagrams

Probability histogram and bar chart





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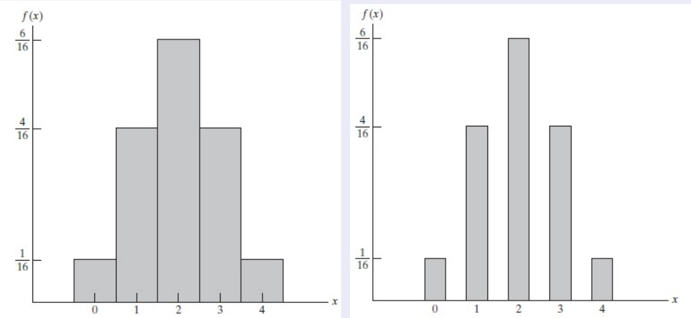

Question What is the difference between histogram and bar chart?

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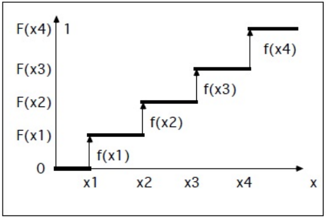

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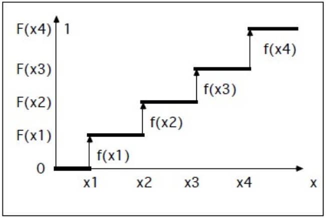

Question Can the pdf be obtained from cdf?