Proability and Statistics

MA20205

Bibhas Adhikari

Autumn 2022-23, IIT Kharagpur

Lecture 14
October 18, 2022

Function of a random variables

Theorem

Let X, Y be continuous random variables with joint pdf $f(x, y)$. Let $U = P(X, Y)$ and $V = Q(x, y)$ be functions of X and Y. If the functions $P(x, y)$ and $Q(x, y)$ have single values inverses, i.e. $X = R(U, V)$ and $Y = R(U, V)$, then the joint density $g(u, v)$ of U, V is given by

$$g(u, v) = |J| f(R(U, V), S(u, v)),$$

where J is the Jacobian given by

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

Function of a random variables

Consequences

1. Let $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$. If X, Y are independent then

$$X + Y \sim \text{Pois}(\lambda_1 + \lambda_2)$$

Function of a random variables

Consequences

1. Let $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$. If X, Y are independent then

$$X + Y \sim \text{Pois}(\lambda_1 + \lambda_2)$$

2. Let the joint density of X, Y be $f(x, y)$. Then

$$f_{X+Y}(v) = \int_{-\infty}^{\infty} f(u, v-u)du$$

Function of a random variables

Consequences

1. Let $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$. If X, Y are independent then

$$X + Y \sim \text{Pois}(\lambda_1 + \lambda_2)$$

2. Let the joint density of X, Y be $f(x, y)$. Then

$$\begin{aligned} f_{X+Y}(v) &= \int_{-\infty}^{\infty} f(u, v-u)du \\ f_{XY}(v) &= \int_{-\infty}^{\infty} \frac{1}{|u|} f(u, \frac{u}{v})du \end{aligned}$$

Function of a random variables

Consequences

1. Let $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$. If X, Y are independent then

$$X + Y \sim \text{Pois}(\lambda_1 + \lambda_2)$$

2. Let the joint density of X, Y be $f(x, y)$. Then

$$\begin{aligned} f_{X+Y}(v) &= \int_{-\infty}^{\infty} f(u, v-u)du \\ f_{XY}(v) &= \int_{-\infty}^{\infty} \frac{1}{|u|} f(u, \frac{u}{v})du \\ f_{\frac{X}{Y}}(v) &= \int_{-\infty}^{\infty} |u| f(u, vu)du. \end{aligned}$$

Function of a random variables

Consequences

1. Let $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$. If X, Y are independent then

$$X + Y \sim \text{Pois}(\lambda_1 + \lambda_2)$$

2. Let the joint density of X, Y be $f(x, y)$. Then

$$\begin{aligned} f_{X+Y}(v) &= \int_{-\infty}^{\infty} f(u, v-u)du \\ f_{XY}(v) &= \int_{-\infty}^{\infty} \frac{1}{|u|} f(u, \frac{u}{v})du \\ f_{\frac{X}{Y}}(v) &= \int_{-\infty}^{\infty} |u| f(u, vu)du. \end{aligned}$$

What happens when X, Y are independent?

Function of a random variables

Consequences

1. Let $X \sim \text{Pois}(\lambda_1)$ and $Y \sim \text{Pois}(\lambda_2)$. If X, Y are independent then

$$X + Y \sim \text{Pois}(\lambda_1 + \lambda_2)$$

2. Let the joint density of X, Y be $f(x, y)$. Then

$$\begin{aligned} f_{X+Y}(v) &= \int_{-\infty}^{\infty} f(u, v-u)du \\ f_{XY}(v) &= \int_{-\infty}^{\infty} \frac{1}{|u|} f(u, \frac{u}{v})du \\ f_{\frac{X}{Y}}(v) &= \int_{-\infty}^{\infty} |u| f(u, vu)du. \end{aligned}$$

What happens when X, Y are independent?

Question What is the pdf of sum of two independent standard normal rvs?