

L-9

Probability & Statistics

Exponential
~~Geometric~~ Distribution.

The r.v. W is described as

↓
waiting time in Poisson
process before occurrence
of an event

$$\begin{aligned} F(w) &= P(W \leq w) \\ &= 1 - P(W \geq w) \\ &= 1 - P(\text{no occurrence} \\ &\quad \text{during } [0, w]) \\ &= 1 - e^{-\lambda w} \end{aligned}$$

$$F'(w) = f(w) = \lambda e^{-\lambda w}.$$

Setting $\theta = \frac{1}{\lambda}$

pdf of Exponential distribution.

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, \quad 0 \leq x < \infty$$

and $\theta > 0$

MGF of exponential distribution.

$$M(t) = \int_0^{\infty} e^{tx} \frac{1}{\theta} e^{-x/\theta} dx$$

$$= \frac{1}{1-\theta t}, \quad t < \frac{1}{\theta}$$

$$M'(t) = \frac{\theta}{(1-\theta t)^2}, \quad M''(t) = \frac{2\theta^2}{(1-\theta t)^3}$$

$$\mu = M'(0) = \theta \text{ and}$$

$$\sigma^2 = M''(0) - [M'(0)]^2 = \theta^2$$

So if λ is the mean number of occurrences in the unit interval, then $\theta = \frac{1}{\lambda}$ is the mean waiting time for the first occurrence.

