# Probability & Statistics.

L-5

Let $X$ be a discrete random variable

moment generating fn. (mgf)

$$= E(e^{tX})$$

$$= \sum_{x_i \in S_x} e^{t x_i} f(x_i)$$

$$= \sum_{x \in S} e^{tx} f(x)$$

$< \infty$, for some $-h < t < h$

$\Rightarrow$ mgf exists.

Idea: mgf uniquely determines the corresponding pmf.

let $S = \{s_1, s_2, \dots\}$

Suppose $X$ & $Y$ are two random variables on $S$, ~~and then~~ $f(x)$, $g(y)$ are the corresponding pmfs.

If mgfs are same,

$$\sum_i e^{ts_i} f(s_i) = \sum e^{ts_i} g(s_i)$$

$$\Rightarrow e^{ts_1} f(s_1) + e^{ts_2} f(s_2) + \dots$$

$$= e^{ts_1} g(s_1) + e^{ts_2} g(s_2) + \dots, \text{ for some } 't'$$

why?

$$\Rightarrow \boxed{f(s_i) = g(s_i)} \quad \forall 'i'.$$

Q. Let $M(t) = \dfrac{e^{t}/2}{1-e^{t}/2}$, $t < \ln 2$

be mgf of a r.v. X. Then find out the corresponding pmf.

Ans. $f(x) = \left(\frac{1}{2}\right)^x$.

(Hint) expand M(t) in terms of a series $\left( \left[1 - \frac{e^t}{2}\right]^{-1} = ? \right)$

$$\frac{e^t}{2}\left(1 + \frac{e^t}{2} + \frac{e^{2t}}{2^2} + \cdots\right)$$

$$= (e^t)\left(\tfrac{1}{2}\right) + (e^{2t})\left(\tfrac{1}{2}\right)^2 + (e^{3t})\left(\tfrac{1}{2}\right)^3 + \cdots$$