L3

# Probability & Statistics

Office Hours – ~~5–6~~ 6–7 pm on Tuesdays
N303, Dept. of Maths.

---

Random Variable : $(S, \mathcal{A}, P)$

$X : S \to \mathbb{R}$
which associates numbers to outcomes.

Exp. Toss a coin $S = \{H, T\}$

$X(H) = 0$ | $Y(H) = a$
$X(T) = 1$ | $Y(T) = b$
~~$a \neq b$~~

many ways to define such a fn.

$f(x) = P(X = x) = P\{w \in S \mid X(w) = x\}$.
$x \in \mathbb{R}$

$P(a \leq X \leq b)$

$f : \mathbb{R} \longrightarrow [0,1]$

Assume $S$ is <u>countably</u>. in

$|S|$ finite or $|S| = |\mathbb{N}|$

Then $f(x) = \underline{P(X = x)}$ is called a pmf or probability 'mass' fn associated with $X$.

Q. ~~Can~~ Does any fn. $f : \mathbb{R} \to [0,1]$ represent a pmf of some random variable?

<u>Defn.</u> (pmf) A fn. '$f$' is a pmf corresponding to $X$

(1) $f(x) > 0$, $x \in S_X$ — $S_X$ = the range set of $X$

(2) $\displaystyle\sum_{x \in S_X} f(x) = 1$

(3) $A \subset S_X$ then,
$P(X \in A) = \displaystyle\sum_{x \in A} f(x)$

$$(X \in A) = \{ w \in S \mid X(w) \in A \}.$$

If $S$ is countably infinite then the corresponding $X$ is called a "discrete random variable".

Distribution fn. of a r.v.

$$F(x) = P(X \leq x)$$

cumulative distribution fn.

Uniform distribution.

$$f(x) = \frac{1}{m}, \quad x = 1, 2, \dots, m.$$

$$S = \{ e_1, \dots, e_m \}$$

$$X : S \to \mathbb{R}$$

$$X(e_i) = i$$

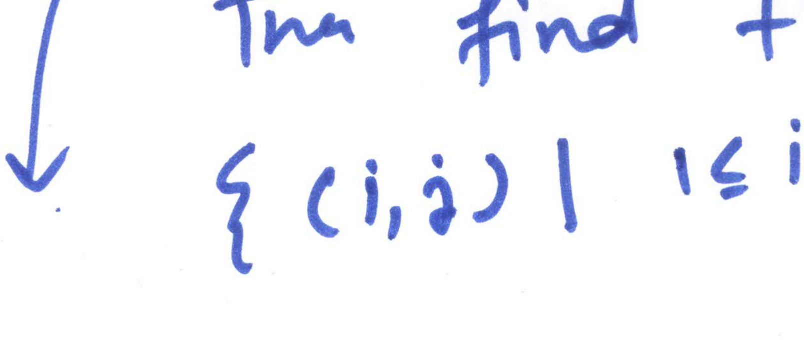
$$F(x) = P(X \le x) = \begin{cases} 0 & , x < 1 \\ \frac{k}{m} & , k \le x < k+1 \\ 1 & , x \ge m \end{cases}$$

<u>Exp.</u> Roll a fair four-sided die twice. Let X be the maximum of the two outcomes. Then find $f(x)$.

$$\{(i,j) \mid 1 \le i \le 4,\ 1 \le j \le 4\}$$

<u>Ans.</u>

$$f(x) = \begin{cases} \frac{2x-1}{16} & , x = 1,2,3,4 \\ 0 & , \text{otherwise}. \end{cases}$$




Bar_diagram.