Probability & Statistics.

L2

Probability model $(S, \mathcal{A}, P)$

$$P : \mathcal{A} \to [0, 1]$$

'Set function'

Q. Can any fn. be a probability measure?

$$P(A) \in [0, 1]$$

Defn. It is <u>non-negative</u> real valued function s.t.

1. $P(A) \geq 0$, $A \subseteq S$
2. $P(S) = 1$
3. if $A_1, \ldots, A_K$ are events and $A_i \cap A_j = \phi$, $i \neq j$ then $P(A_1 \cup \ldots \cup A_K) = \sum_{i=1}^{K} P(A_i)$

In general $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

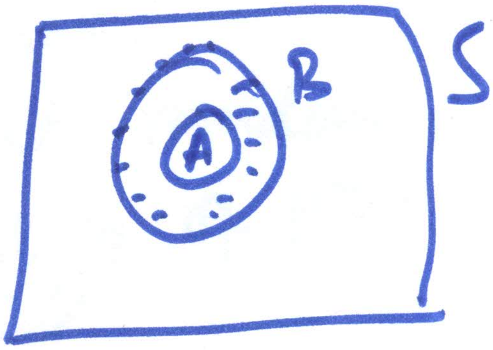
whenever $A_k \cap A_j = \phi$, $k \neq j$

Properties of 'P'

① $P(A) = 1 - P(A^c)$

② $P(\phi) = 0$

③ $A \subseteq B$, then $P(A) \leq P(B)$.



④ $P(A) \leq 1$

⑤ $A, B, C \subseteq S$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Exp. of a prob. measure.

$$S = \{e_1, \ldots, e_m\}$$

$P(\{e_i\}) = \frac{1}{m}$, the outcomes are "equally likely".