L-13 Probability & statistics.

(X, Y) X, Y are discrete random variables

joint pmf.

range of $X = S_x = \{x_1, x_2, \dots\}$ range of $Y = S_y = \{y_1, y_2, \dots\}$

$$f(x,y) = f(x_i, y_j) = P(X = x_i, Y = y_j)$$

Let (X, Y) be a pair of continuous random variables.

$f(x,y) \rightarrow$ joint pdf of (X,Y) has to satisfy the following properties.

1. $f(x,y)$ is integrable
2. $f(x,y) \geq 0$
3. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$
4. For any $A \subseteq S_X \times S_Y$, $P((X,Y) \in A) = \iint_A f(x,y)\,dx\,dy$

marginal pdfs of X & Y

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\, dy, \quad x \in S_x$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\, dx, \quad y \in S_y$$

$\mu_x, \ \mu_y, \ \sigma_x^2, \ \sigma_y^2$

Exp. $f(x,y) = \left(\frac{4}{3}\right)(1 - xy), \quad 0 \le x \le 1, \ 0 \le y \le 1$

Find. $f_X(x), \ f_Y(y), \ \mu_x, \ \mu_y, \ \sigma_x^2, \ \sigma_y^2.$

$P(Y \le X/2) = ?$

Ans

$f_X(x) = \left(\frac{4}{3}\right)\left(1 - \frac{x}{2}\right), \quad 0 \le x \le 1$

$\mu_x = 4/9, \quad \sigma_x^2 = \frac{13}{162}$

$$P(Y \le X/2) = \int_0^1 \int_0^{x/2} f(x,y)\, dy\, dx = \frac{7}{24}.$$

Exp. $f(x,y) = 1, \quad x \le y \le x+1,$

$0 \le x \le 1$

Find, $f_X(x)$, $f_Y(y)$, μ_x, μ_y,

σ_x^2, σ_y^2, $Cov(X,Y)$

$\rho =$

H.W. $|\rho| \le 1.$

Bivariate Normal distribution

$$X \sim N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

↑ bell shape

pdf of $N(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2)$

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left[-\frac{q(x,y)}{2}\right]$$

where

$$q(x,y) = \frac{1}{1-\rho^2}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\frac{(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]$$