

L-13

Probability & Statistics.

(X, Y) X, Y are discrete
joint pmf. random variables.

range of $X = S_x = \{x_1, x_2, \dots\}$
range of $Y = S_y = \{y_1, y_2, \dots\}$

$$f(x, y) = f(x_i, y_j) = P(X = x_i, Y = y_j)$$

Let (X, Y) be a pair of continuous random variables.

$f(x, y) \rightarrow$ joint pdf of (X, Y)
has to satisfy the following
properties.

① $f(x, y)$ is integrable

② $f(x, y) \geq 0$

③ $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$

④ For any $A \subseteq S_x \times S_y$, $P((X, Y) \in A) = \iint_A f(x, y) dx dy$

marginal pdfs of X & Y

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y) dy, \quad x \in S_x$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y) dx, \quad y \in S_y$$

$$\mu_x, \mu_y, \sigma_x^2, \sigma_y^2$$

Exp. $f(x,y) = \left(\frac{4}{3}\right)(1-xy), \quad \begin{matrix} 0 \leq x \leq 1 \\ 0 \leq y \leq 1 \end{matrix}$

Find. $f_X(x), f_Y(y), \mu_x, \mu_y, \sigma_x^2, \sigma_y^2$.

$P(Y \leq x/2) = ?$

Ans

$$f_X(x) = \left(\frac{4}{3}\right)\left(1 - \frac{x}{2}\right), \quad 0 \leq x \leq 1$$

$$\mu_x = 4/9, \quad \sigma_x^2 = \frac{13}{162}$$

$$P(Y \leq x/2) = \int_0^1 \int_0^{x/2} f(x,y) dy dx = \frac{7}{24}$$

Exp. $f(x, y) = 1, \quad x \leq y \leq x+1,$
 $0 \leq x \leq 1$

Find, $f_x(x), f_y(y), \mu_x, \mu_y,$
 $\sigma_x^2, \sigma_y^2, \text{Cov}(x, y)$

$\rho =$

H.W. $|\rho| \leq 1.$

Bivariate Normal distribution

$X \sim N(\mu, \sigma^2)$

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

↑ bell shape

