L-12

Probability & Statistics.

pair of random variables (X, Y).

joint pmf.

$$P(X=x, Y=y) = f(x,y).$$

f(x,y) is a pmf if

1. $f(x,y) \geq 0$
2. $\sum_{x \in S_X} \sum_{y \in S_Y} f(x,y) = 1$
3. $A \subseteq S_X \times S_Y$,

$$P((X,Y) \in A) = \sum_{(x,y) \in A} f(x,y).$$

marginal pmf.

$$f_X(x,y) = \sum_{y \in S_Y} f(x,y)$$

μ_X , μ_Y

Expectation of a function $u(X,Y)$

$$E\big(u(X,Y)\big) = \sum_{x,y \in S_X \times S_Y} u(x_i, y_j)\, f(x_i, y_j)$$

Covariance.

$$\text{Cov}(X,Y) = E\big(\underbrace{(X-\mu_X)(Y-\mu_Y)}_{u(X,Y)}\big) = \sigma_{XY}.$$

Correlation coefficient:

$$\rho = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}.$$

Note: covariance can not be computed from the marginal distributions.

Note:

$$\mathrm{cov}(X,Y) = E\left((X-\mu_x)(Y-\mu_y)\right)$$

$$= E(XY) - \mu_x E(Y) - \mu_y E(X) + \mu_x \mu_y$$

$$= E(XY) - \mu_x \mu_y \quad \text{---} \quad (*)$$

Since $\rho = \dfrac{\mathrm{cov}(X,Y)}{\sigma_x \sigma_y}$,

from $(*)$ we obtain

$$\boxed{E(XY) = \mu_x \mu_y + \rho\, \sigma_x \sigma_y .}$$

Exp.

$$f(x,y) = \frac{x+2y}{18}, \quad \begin{array}{l} x = 1,2 \\ y = 1,2 \end{array}$$

Compute: $f_X(x)$, $f_Y(y)$, μ_x, μ_y, σ_x^2, σ_y^2, $\mathrm{cov}(X,Y)$, ρ.