L-11

Probability & Statistics.

Bivariate ~~random~~ probability distributions.

E, S

random experiment

sample space

one measurement:

$X : S \rightarrow \mathbb{R}$

$\cancel{s} \mapsto X(s) = x$

$P(X = x)$, $P(X \leq x)$

pmf/pdf

cdf

two measurements

Exp. $X \rightarrow$ measures height of a student

$Y \rightarrow$ measures weight of a student.

S_X = rangeset of X, S_Y = range set of Y

Q. How X is related to Y.

Exp. $X \to$ rank in school before joining IIT

$Y \to$ score in JEE.

Suppose $Z \to$ measures your CGPA in first year.

$z = u(x, y)$?

We want to analyze a pair of random variables (X, Y).

$$(x, y) \in S_X \times S_Y$$

Assume X, Y are both discrete r.v.s

<u>Joint probability mass function</u>

$$f(x, y) = p(x, y) = P(X = x, Y = y)$$

Then $f(x, y)$ must satisfy

(i) $0 \le f(x, y) \le 1$

(ii) $\sum_{(x,y) \in S_X \times S_Y} f(x, y) = 1$

(iii) $A \subseteq S_X \times S_Y$, $P((X, Y) \in A) = \sum_{(x,y) \in A} f(x,y)$

Ex. Roll a pair of fair dice.

$S = \{ (x,y) \mid x, y \in \{1,2,\dots,6\} \}$.

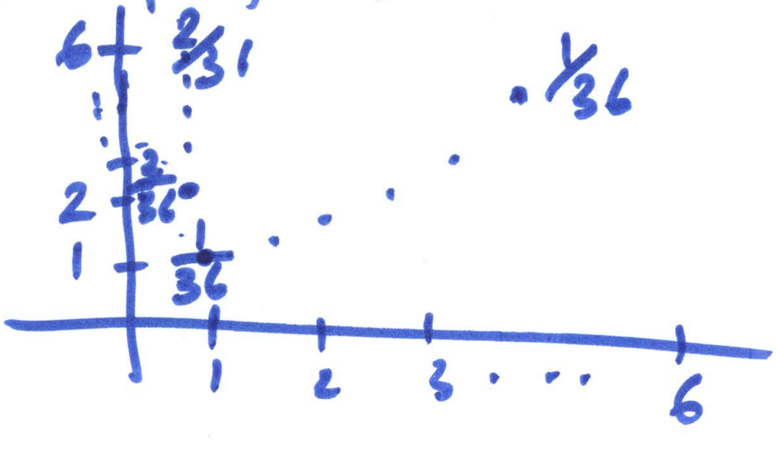
Let X - denote the smaller
Y - denote the larger value.

(For example, $(2,3) \in S$

$X((2,3)) = 2$

$Y((2,3)) = 3$

Write the joint pmf of X & Y



$$f(x,y) = p(x,y) = \begin{cases} \frac{1}{36}, & 1 \le x = y \le 6 \\ \frac{2}{36}, & x \neq y \end{cases}$$

Q. Can we find pmf of X and Y from the joint pmf $f(x,y)$.

Marginal probability mass function.

Let the joint pmf $f(x,y)$

$$f_X(x) = \sum_y f(x,y)$$

$$= \sum_y P(X=x, Y=y)$$

$$f_Y(y) = \sum_x f(x,y)$$

$$= \sum_x P(X=x, Y=y)$$

Independent pmf

$$f(x,y) = f_X(x)\, f_Y(y) \quad \forall (x,y) \in S$$

otherwise X, Y are called dependent.

Ex.

Joint pmf of X & Y

$$f(x,y) = \frac{xy^2}{13},$$

$(x,y) \in \{(1,1),(1,2),(2,2)\}$

Calculate $f_X(x)$, $f_Y(y)$ and decide if X & Y are independent!

Ans. dependent.

Expectation of (X,Y)

Let $U(X,Y)$ be a function of (X,Y). Then the expectation of (X,Y)

$\cancel{E((X,Y)) =}$

$$E(U(X,Y)) = \sum_{x_i} \sum_{y_j} u(x_i,y_j)\, f(x_i,y_j)$$

$\mu_X = E(X)$, $\mu_Y = E(Y)$

$\sigma_X^2 = E((X-\mu_X)^2)$

$\sigma_Y^2 = E((Y-\mu_Y)^2)$.

(a) $E[(X-\mu_X)(Y-\mu_Y)]$ → think.

Mid Sem Syllabus - up to Lecture 9