

Lec-10

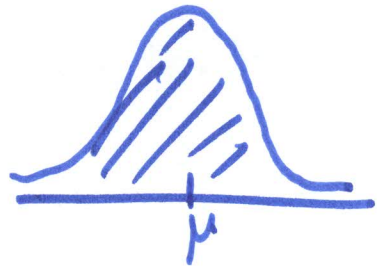
Probability & Statistics.

Normal distribution.

pdf: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], -\infty < x < \infty$

$\sigma > 0$ and $-\infty < \mu < \infty$.

$$X \equiv N(\mu, \sigma^2)$$



MAF of X.

$$M(t) = \int_{-\infty}^{\infty} \frac{e^{tx}}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left\{-\frac{1}{2\sigma^2} [x^2 - 2(\mu + \sigma^2 t)x + \mu^2]\right\} dx$$

$$x^2 - 2(\mu + \sigma^2 t)x + \mu^2 = [x - (\mu + \sigma^2 t)]^2 - 2\mu\sigma^2 t + \sigma^4 t^2$$

$$= \exp\left(\frac{2\mu\sigma^2 t + \sigma^4 t^2}{2\sigma^2}\right) \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2} [x - (\mu + \sigma^2 t)]^2\right] dx$$

$$= \exp\left[\frac{2\mu\sigma^2 t + \sigma^4 t^2}{2\sigma^2}\right] = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$$

$$E(X) = M'(0) = \mu$$

$$\text{Var}(X) = M''(0) - [M'(0)]^2 \\ = \sigma^2.$$

~~The~~ If $Y = N(0, 1)$ it is called standard normal distribution. and the cdf of Y is

$$\Phi(y) = P(Y \leq y) \\ = \int_{-\infty}^y \frac{1}{\sqrt{2\pi}} e^{-w^2/2} dw.$$

Exp. If $Z \approx N(0, 1)$, find

$$P(Z \leq 1.24) = \Phi(1.24) = 0.8925$$

$$P(1.24 \leq Z \leq 2.37) = \Phi(2.37) - \Phi(1.24).$$

$$P(-2.37 \leq Z \leq -1.24)$$

m. If $X = N(\mu, \sigma^2)$ then
 $Z = \frac{X - \mu}{\sigma}$ then $Z \approx N(0, 1)$

Pr. cdf. of Z

$$\begin{aligned} P(Z \leq z) &= P\left(\frac{X - \mu}{\sigma} \leq z\right) \\ &= P(X \leq \sigma z + \mu) \\ &= \int_{-\infty}^{\sigma z + \mu} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x - \mu)^2}{2\sigma^2}\right] dx \end{aligned}$$

Let $w = \frac{x - \mu}{\sigma}$

$$= \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-w^2/2} dw$$

↳ which is $N(0, 1)$

