

**Probability and Statistics**  
**Assignment No. 6**

1. Let  $(X, Y)$  have the joint pmf

|                  |      |      |      |
|------------------|------|------|------|
| $Y \backslash X$ | -1   | 0    | 1    |
| -2               | 1/6  | 1/12 | 1/6  |
| 1                | 1/6  | 1/12 | 1/6  |
| 2                | 1/12 | 0    | 1/12 |

Find the joint pmf of  $(U, V)$  where  $U = |X|$  and  $V = Y^2$ .

2. Projectiles are fired at the origin of an  $XY$  – coordinate system. Assume that the point which is hit, say  $(X, Y)$ , consists of a pair of independent standard normal r.v.'s. For two projectiles fired independently of one another, let  $(X_1, Y_1)$  and  $(X_2, Y_2)$  represent the points which are hit and  $Z$  be the distance between them. What is the distribution of  $Z^2$  ?
3. Let  $X_1$  and  $X_2$  be independent r.v.'s each with negative exponential distribution with pdf  $\lambda \exp\{-\lambda x\}$ ,  $x > 0$ . Find the joint and marginal distributions of  $Y_1 = X_1/X_2$  and  $Y_2 = X_1 + X_2$ .
4. Let  $X_1, X_2$  be i.i.d.  $N(0,1)$  and  $Y_1 = X_1^2 + X_2^2$ ,  $Y_2 = X_1/X_2$ . Are  $Y_1, Y_2$  independent?
5. Let  $X_1$  and  $X_2$  have independent gamma distributions with parameters  $(n_1, \lambda)$  and  $(n_2, \lambda)$ . Find the distributions of  $Y = X_1 / (X_1 + X_2)$ . Is  $Y$  independent of  $Z = X_1 + X_2$ ? Is  $Z$  independent of  $U = X_1/X_2$ ?
6. Let  $X_1, X_2, X_3$  be independent exponential random variables with the probability density  $f(x) = e^{-x}$ ,  $x > 0$ . Define random variables  $Y_1, Y_2$  and  $Y_3$  as  $Y_1 = X_1 + X_2 + X_3$ ,  $Y_2 = \frac{X_1 + X_2}{X_1 + X_2 + X_3}$ ,  $Y_3 = \frac{X_1}{X_1 + X_2}$ .

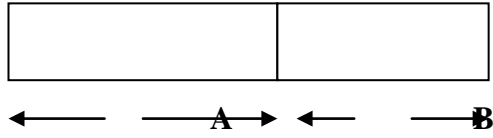
Find the joint and marginal densities of  $Y_1, Y_2$  and  $Y_3$ . Are they independent?

7. Suppose independent random variables  $Y_1, Y_2, Y_3$  are such that  $Y_1 = \ln X_1 \sim N(4, 1)$ ;  $Y_2 = \ln X_2 \sim N(3, 1)$ ;  $Y_3 = \ln X_3 \sim N(2, 0.5)$ . Find the distribution and the median of  $W = e^2 X_1^2 X_2^{1.5} X_3^{1.28}$ . Determine  $L$  and  $R$  such that  $P(L \leq W \leq R) = 0.90$ .
8. Let  $(X, Y)$  have bivariate normal distribution with density function

$$f(x, y) = \frac{1}{\pi\sqrt{3}} e^{-\frac{2}{3}(x^2 - xy + y^2)}, \quad -\infty < x, y < \infty.$$

Find the correlation coefficient between  $X$  and  $Y$ ,  $P(-1 < X < 1 | Y=1)$ ,  $V(2X + 3Y)$  and  $P(-5 < 2X + 3Y < 8)$ .

9. A straight rod consists of two sections **A** and **B**, each of which is manufactured independently on a different machine. The length (in inches) of section **A** is normally distributed with mean **20** and variance **0.03** and the length of section **B** is normally distributed with mean **14** and variance **0.01**. The rod is formed by joining the two sections together as shown below:



Suppose that the rod can be used in the construction of an airplane wing if its total length is between **33.6** to **34.4** inches. What is the probability that the rod can be used in the construction?