

**Probability and Statistics**  
**Assignment No. 4**

1. Let  $X$  be a continuous random variable with density function given by

$$f_X(x) = \begin{cases} 2(x+1)/9, & -1 < x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find the density function of  $Y = X^2$ .

2. Let  $X$  be continuous random variable with density given by

$$f_X(x) = \begin{cases} x/2, & 0 \leq x < 1 \\ 1/2, & 1 < x \leq 2 \\ (3-x)/2, & 2 < x \leq 3 \end{cases}$$

Find the density of  $Y = (X - 3/2)^2$ .

3. Let  $X$  be a random variable with density function given by

$$f_X(x) = \begin{cases} 2x/\pi^2, & 0 < x < \pi \\ 0, & \text{otherwise.} \end{cases}$$

Find the distribution of  $Y = \sin X$ .

4. Let  $X \sim \text{Bin}(n, p)$ . Find the p.m.f. of each of the following functions of  $X$  :

(a)  $Y_1 = 3X + 4$ ; (b)  $Y_2 = X - 3$ ; (c)  $Y_3 = X^2 + 2$ ; (d)  $Y_4 = \sqrt{X}$ .

5. Let  $X \sim \text{Beta}(a, b)$ . Find the distributions of  $Y_1 = 1/(1 + X)$  and  $Y_2 = 1 - X$ .

6. Let  $C$  denote the temperature in degree Celsius to which a computer will be subjected to in the field. Assume that  $C$  is uniformly distributed over the interval  $(15, 21)$ . Let  $F$  denote the field temperature in degrees Fahrenheit so that  $F = (9/5)C + 32$ . Find the density of  $F$ .

7. Let  $X$  denote the velocity of a random gas molecule. According to the Maxwell-Boltzmann law, the density for  $X$  is given by

$$f_X(x) = cx^2 \text{Exp}\{-\beta x^2\}, \quad x > 0.$$

The kinetic energy of the molecule,  $Y$ , is given by  $Y = (1/2) mX^2$ , where  $m$  is positive. Find the density of  $Y$ .

8. Let  $X$  be a random variable with the pdf

$$f(x) = \begin{cases} \frac{x+1}{4}, & -1 \leq x \leq 1 \\ \frac{3-x}{4}, & 1 \leq x \leq 3. \end{cases}$$

Find the distribution of  $Y = |X|$ .