1. Let X be a continuous random variable with the density function given by
$$f_X(x) = \begin{cases} \dfrac{2(x+1)}{9} & -1 < x < 2 \\ 0 & otherwise \end{cases}$$
Find the density function of $Y = X^2$.

2. Let X be continuous random variable with the density given by
$$f_X(x) = \begin{cases} \dfrac{x}{2} & 0 < x \le 1 \\ \dfrac{1}{2} & 1 < x \le 2 \\ \dfrac{3-x}{2} & 2 < x < 3 \end{cases}$$

Find the density of $Y = \left(X - \dfrac{3}{2}\right)^2$.

3. Let X be a random variable with the density function given by
$$f_X(x) = \begin{cases} \dfrac{2x}{\pi^2} & 0 < x < \pi \\ 0 & otherwise \end{cases}$$
Find the distribution of $Y = \sin X$.

4. Let $X \sim Bin(n, p)$. Find the p.m.f. of each of the following functions of X:

(a) $Y_1 = 3X + 4$; (b) $Y_2 = X - 3$; (c) $Y_3 = X^2 + 2$; (d) $Y_4 = \sqrt{X}$.

5. Let $X \sim Beta(a, b)$. Find the distributions of $Y_1 = \dfrac{1}{1+X}$ and $Y_2 = 1 - X$.

6. Let C denote the temperature in degree Celsius to which a computer will be subjected to in the field. Assume that C is uniformly distributed over the interval $(15, 21)$. Let F denote the field temperature in degrees Fahrenheit so that $F = \dfrac{9}{5}C + 32$. Find the density of F.

7. Let X denote the velocity of a random gas molecule. According to the Maxwell-Boltzmann law, the density for X is given by
$$f_X(x) = cx^2 e^{-bx^2} , \quad x > 0.$$

The kinetic energy of the molecule, is given by $Y = \frac{1}{2}mX^2$, where m is positive. Find the density of Y.

8. Let X be a random variable with the pdf
$$f_X(x) = \begin{cases} \dfrac{x+1}{4} & -1 < x \le 1 \\ \dfrac{3-x}{4} & 1 < x < 3 \end{cases}$$

Find the distribution of $Y = |X|$.

9. Let X be a standard normal random variable and
$$Y = \begin{cases} \dfrac{\sqrt{X}}{2} & X \ge 0 \\ -\sqrt{|X|} & X < 0 \end{cases}$$

Find the pdf of Y.

10. Let X be a discrete random variable with the p.m.f. given by

$$P(X = i) = \frac{1}{4i^2} \ , \quad P(X = i) = \frac{1}{(1+i)^2} \ , i = 1,2,3$$

and $P(X = 0) = \frac{17}{72}$.

Find the distribution of $Y = X^2$ and determine its median.

11. Let X be a continuous random variable with the p.d.f. given by
$$f_X(x) = \begin{cases} k\sqrt{x} & 0 < x < 1 \\ 0 & otherwise \end{cases}$$
where k is a suitable constant. Find the pdf of $Y = \sqrt[4]{X}$.

12. Let X follow a uniform distribution on the interval $(-1, 1)$. Let $Y = -2\log_e |X|$. Find $E(Y)$.