MA 20205 Probability and Statistics
Assignment No. 3

1. Ruby and Mini tied for the first place in a beauty contest. The winner is to be decided by the majority opinion of a panel of three judges chosen at random from a group of seven judges. If four of these judges favour Ruby and three favour Mini, what is the probability that Ruby will be declared the winner.

2. In a precision bombing attack there is a **50%** chance that a bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give at least **99%** chance of completely destroying the target?

3. It is known that any electronic device produced by a certain company will be defective with probability 0.1, independently of any other item. What is the probability that in a sample of three items, at most one will be defective?

4. Let X follow a Binomial distribution with mean 8 and standard deviation 2. Find $P(X \geq 3)$.

5. A communication system consists of n components each of which function independently with probability p. The entire system will be able to operate effectively, if at least one-half of its components function. For what values of p, a 5-component system more likely to operate effectively than a 3-component system?

6. Let the mgf of a random variable X be given by

$$M_X(t) = \frac{2\,e^t}{7-5\,e^t}, \quad t < \log_e\left(\frac{7}{5}\right)$$

Find $P(X > 7 \mid X > 5)$.

7. The diskettes produced by a certain company are defective with probability 0.01, independently of each other. The company sells the diskettes in packs of size 10 and offers a money-back guarantee if more than one of the 10 diskettes in the pack is found to be defective. If you buy 3 packs, what is the probability that at most one pack will be returned?

8. The probability that an experiment has a successful outcome is 0.8. The experiment is to be repeated until three successful outcomes have occurred. Let X be the number of repetitions required in order to have 3 successful outcomes? What is the probability that at least 5 repetitions will be required?
9. An electronic store has twenty TV sets. Five of these have some manufacturing defect. A customer randomly selects four TV sets. Find the probability that the sample will have exactly one defective.

10. The number X of computers that a hardware store sells in a week obeys the Poisson distribution with $\lambda = 2$. The profit on each computer is Rs. 2000.00. If at the time of opening the store 10 computers are in stock (with no replenishment during the daytime), the profit from the sale of the computers during the day is $Y = 2000 \min(X, 10)$. Find the probability distribution of Y.

11. The number of times that an individual contracts cold in a given year is a Poisson random variable with parameter $\lambda = 3$. Suppose that a new drug has been just marketed that reduces the parameter λ to 2 for 75% of the population. For the other 25% of the population the drug has no appreciable effect on the cold. If an individual tries the drug for a year and has no cold in that time, how likely is it that the drug is beneficial for him?

12. A large microprocessor chip contains multiple copies of circuits. If a circuit fails, the chip knows it and knows how to select the proper logic to repair itself. The average number of defects per chip is 300. What is the probability that no more than 4 defects will be found in a randomly selected area that comprises 2% of the total surface area?

13. A boy and a girl decide to meet between 5 and 6 p.m. in a park. They decide not to wait for the other for more than 20 minutes. Assuming arrivals to be independent and uniformly distributed, find the probability that they will meet.

14. Find $Var(X)$, if the mgf of the random variable X is

$$M_X(t) = \begin{cases} \dfrac{e^t(e^{10t} - 1)}{10\,(e^t - 1)}, & \text{if } t \neq 0 \\ 1, & \text{if } t = 0 \end{cases}.$$

15. A contractor has found through experience that the low bid for a job is a uniform random variable on $(\frac{3}{4}C, 2C)$, where C is the contractor's cost estimate (no profit, no loss) of the job. The profit is defined as zero if the contractor does not get the job and as the difference between his bid and his cost estimate C if he gets the job. What should he bid (in terms of C) in order to maximize his expected profit?

16. A small industrial unit has **10** bulbs whose lifetimes are independent exponentially distributed with mean **50** hours. If all the bulbs are used at a time, find the probability that even after **100** hours there are at least two bulbs working.

17. The time to failure in months, X, of the light bulbs produced at two manufacturing plants A and B obeys exponential distribution with means 5 and 2 months respectively. Plant B produces three times as many bulbs as plant A. The bulbs indistinguishable to eye are intermingled and sold. What is the probability that a bulb purchased at random will burn at least 5 months?

18. The motherboard of a new CPU is guaranteed for 6 months. The mean life of a motherboard is estimated to be two years, and the time to failure has an exponential density. The realized profit on a new CPU is Rs. 5,000.00. Including costs of parts and labour the dealer must pay Rs. 2000.00 to repair each failure. Assuming at most one failure in the first 6 months, find the expected profit (in Rs.) per CPU.

19. A series system has n independent components. For i = 1,…, n, the lifetime X_i of the i^{th} component is exponentially distributed with parameter λ_i. If the system has failed before time t what is the probability the failure was caused only by component j (j = 1,…,n).

20. A small shopping mall has five air-conditioner (AC's). The lifetimes of ACs follow independent and identical exponential distributions with mean 100 hours. If all AC's are used simultaneously, find the probability that after 100 hours there are at least two AC's in working condition.

21. The time (in minutes) between arrivals of customers at an ATM machine is exponentially distributed random variable with mean 10 minutes. What is the probability that starting at 9:00 a.m., the third customer will arrive within fifteen minutes?

22. The lead time for orders of diodes from a certain manufacturer is known to have a gamma distribution with a mean of 20 days and a standard deviation of 10 days. Determine the probability of receiving an order within 15 days of placement date.

23. The life (in years) of an electronic equipment is known to follow a gamma distribution. The equipments produced by manufacturer 'A' have mean 4 and variance 8 whereas those produced by manufacturer 'B' have mean 2 and variance 4. An organization procures 75% units of the equipment from 'A' and 25% from 'B'. A unit selected at random is found to be working after 12 years. Find the probability that it was produced by 'A'.

24. A computer lab has three printers. Printer I handles 30% of all jobs and its printing time follows an exponential distribution with mean 3 minutes. Printer II also handles 30% of all jobs and its printing time follows a gamma distribution with mean time 2 minutes and variance 2. Printer III handles remaining 40% of all the jobs and its printing time follows a uniform distribution between 0 and 4 minutes. Find the probability that a randomly selected job will be printed in less than one minute.

25. The lifetime X in hours of a component is modelled as a Weibull distribution with $\beta = 2$. Starting with a large number of components it is observed that 15% of the components that have lasted 90 hours fail before 100 hours. Determine the parameter α. Further determine the probability that a component is working after 80 hours.