MA 20205 Probability and Statistics

Assignment No. 2

1. Let X be a continuous random variable with the probability density function

$$
\begin{aligned}
f(x) &= x^2, \quad 0 < x \le 1, \\
&= \frac{c}{x^2}, \quad 1 < x < \infty, \\
&= 0, \quad \text{elsewhere.}
\end{aligned}
$$

   Determine c and cumulative distribution function of X. Further find $E(X), Var(X)$ and the median of X. Find $P(0.5 < X < 2)$ and $P(X > 3)$.

2. Let X be a random variable with the cdf given by

$$
F(x) = \begin{cases} 0, & x < 0 \\ \dfrac{x}{4}, & 0 \le x < 1 \\ \dfrac{x+1}{4}, & 1 \le x < 2. \\ \dfrac{11}{12}, & 2 \le x < 3 \\ 1, & x \ge 3 \end{cases}
$$

   Find $P\left(\frac{1}{2} < X < \frac{5}{2}\right)$, $P(1 < X < 3)$, $E(X)$, $V(X)$ and the median of X.

3. A guinea pig either dies (D) or survives (S) in the course of a surgical experiment. The experiment is to be performed first with two guinea pigs. If both survive, no further trials are to be made. If exactly one guinea pig survives, one more guinea pig is to undergo the experiment. If both guinea pigs die, two additional guinea pigs are to be tried. Assuming that the trials are independent and that the probability of survival in each trial is 2/3 find the probability distributions of the number of survivals and the number of deaths.

4. Suppose that a particle is equally likely to release one, two or three other particles and suppose that these second generation particles are, in turn, equally likely to release one, two or three third generation particles. Find the pmf of the number of third generation particles.

5. Let the distribution of scores on an IQ test have mean 100 and standard deviation 16. Use Chebyshev's inequality to show that the probability of a student having IQ above 148 or below 52 is at most 1/9.

6. Find cdf, mean, variance and the median of X if X is a continuous random variavle with pdf given by

$$f(x)=\begin{cases} \frac{x}{2}, & 0 \le x \le 1 \\ \frac{1}{2}, & 1 < x \le 2 \\ \frac{3-x}{2}, & 2 < x \le 3 \\ 0, & \text{otherwise} \end{cases}.$$

7. Let X be a continuous random variable with

$$f(x)=\begin{cases} \frac{k}{4}, & 0 < x < 1 \\ \frac{x}{2}, & 1 \le x \le 2 \\ \frac{1-k}{4}, & 2 < x < 3 \end{cases}.$$

Determine the values of k for which $f(x)$ is a density function. Find the cumulative distribution function of X and use it to determine the median M of X. Show that $\sqrt{2} \le \mathrm{M} \le \sqrt{3}$. Find the measures of skewness and kurtosis and interpret them for valid values of k.

8. To determine whether or not they have a certain blood disease, 100 people are to have their blood tested. However, rather than testing each individual separately, it has been decided first to group the people in batches of 10. The blood samples of the 10 people in each batch will be pooled and analyzed together. If the test is negative, one test will suffice, else each of the ten people will be individually tested. Assuming the incidence of disease to be 1% for the population, find the expected number of tests required for each batch.

9. A certain community is composed of m families, n_i of which have i children, $\sum_{i=1}^{r} n_i = m$. If one of the families is randomly chosen, let X denote the number of children in that family. Find $E(X)$. If one of the $\sum_{i=1}^{r} i\, n_i$ children is randomly

chosen, let Y denote the total number of children in the family of that child. Find $E(Y)$. Show that $E(Y) \geq E(X)$.

10. Let X be a discrete random variable with $p_X(1)=\frac{1+3d}{4}$, $p_X(2)=\frac{1-d}{4}$, $p_X(3)=\frac{1+2d}{4}$ and $p_X(4)=\frac{1-4d}{4}$. For what values of d, does $p_X(x)$ describe a valid probability mass function? Further determine the value of d for which $Var(X)$ is a minimum.

11. Let X denote the number of accidents in a factory per week having p.m.f.

$$p(x)=\frac{k}{(x+1)(x+2)},\ \ x=0,1,2,\ldots$$

Find the value of k, cdf of $X, E(X)$ and the median of X.

12. The number of items produced in a factory during a week is a random variable with mean 50 and standard deviation 5. Using Chebyshev's inequality find the minimum probability that this week's production will be between 40 and 60 ?

13. Let X be a random variable with m.g.f. given by $\mathrm{M}_{\mathrm{X}}(t) = \frac{(3+2\mathrm{e}^{t})^{4}}{625}$.

Find the mean, variance and $P(X \leq 1)$. Is the distribution positively or negatively skewed?

14. A fair coin is tossed until it shows up the same face twice in succession. Let X be the number of tosses to complete the experiment. Find the probability mass function of X. What is the probability that an even number of tosses is required to end the experiment? Given that the experiment ended in an even number of trials, what is the probability that it ended on the 6th trial?

15. Players A and B each possesses a biased dice. The dice of Player A, when tossed, has probability of showing upper face k proportional to k, for $k=1,2,\ldots,6$. Similarly the dice of Player B, when tossed, has probability of showing upper face k proportional to k^2, for $k=1,2,\ldots,6$. Both the players toss their dice independently and simultaneously. Let X denote the absolute difference of the numbers on the upper faces of the two dice. Find the probability distribution of X.