**MA 20205 Probability and Statistics**
**Assignment No. 1**

1. If 7 balls are placed at random into 7 cells, find the probability that exactly one cell remains empty.

2. Let event E be independent of events F, $F \cup G$ and $F \cap G$. Show that E is independent of G.

3. A pair of dice is rolled until a sum of 4 or an odd sum appears. Find the probability that a 4 appears first.

4. In a certain university, 50% of the faculty members own a desktop computer, 25% own a laptop and 10% own both a desktop and a laptop. If a faculty member is randomly chosen, find the probability that he/she owns a desktop or a laptop but not both.

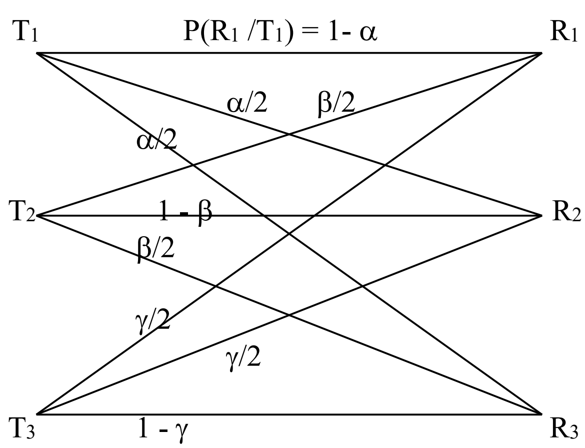
5. A survey of people in given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is 0.006, what is the probability of death due to lung cancer given that a person is a smoker?

6. A box contains *n* balls marked from 1 to *n*. Two balls are drawn in succession with replacement. Find the probability that number on the balls are consecutive integers (ignore the order of balls).

7. Let $A$ and $B$ be two events with $P(A) < 1, P(B) > 0$ and $P(A \mid B) = 1$. Determine $P(B^C \mid A^C)$.

8. If $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$, find $P(B \mid A \cup B^c)$.

9. Consider a trinary communication channel whose channel diagram is shown below:



For i = 1,2,3, let $T_i$ denote the event "Digit i is transmitted" and let $R_i$ denote the event "Digit i is received". Assume that a 3 is transmitted three times more frequently than a 1, and a 2 is transmitted twice as often as 1. (i) If a one has been received, what is the probability that a 1 was sent? (ii) Find the probability of a transmission error. (iii) Find the probability that digit i is received for i = 1,2,3.

10. Which of the following statements is true?
    (i) $P(A) = 0.3$, $P(B) = 0.7$, $P(A \cup B) = 0.5$, $P(A \cap B) = 0.5$.
    (ii) $P(A) = 0.5$, $P(A \cup B) = 0.7$, A and B are independent, $P(B) = 0.4$.
    (iii) $P(A) = 0.2$, $P(A \cup B) = 0.9$, A and B are disjoint, $P(B) = 0.6$.
    (iv) none of these.

11. Four players A, B, C and D are distributed thirteen cards each at random from a complete deck of 52 cards. What is the probability that player C has all four kings?

12. An Two Year M.Sc. student has to take 5 courses a semester for 4 semesters. In each course he/she has a probability 0.5 of getting at least an 'A' grade. Assuming the grades to be independent in each course, what is the probability that he/she will have all 'A' or above grades in at least one semester.

13. If $P(A) > 0$, show that $P(A \cap B \mid A) \geq P(A \cap B \mid A \cup B)$.

14. Let $S = \{1, 2, \ldots, n\}$ and suppose that A and B are, independently, equally likely to be any of the $2^n$ subsets (including) the null set and S itself) of S. Let X denote the number of elements of B. Find $P(X = i)$, $P(A \subset B \mid X = i)$, $P(A \subset B)$ and deduce that $P(A \cap B = \phi) = \left(\frac{3}{4}\right)^n$.

15. A question paper consists of six True-False and four multiple choice (A, B, C, D) questions. Each question carries one mark for the correct answer and zero for wrong answer. Assume that an unprepared student answers all questions independently with guess. What is the probability that he/she will score at least 8 marks?

16. Boys and girls are equally likely to qualify an examination. If 2n students qualify, what is the probability that more girls qualify than boys?

17. Suppose *n* men take part in a get-together in a hall. When they enter into the hall they put off their hats and keep on a table. While leaving the hall they pick up a hat at random. What is the probability that no one will get back his own hat? What is the limiting probability when *n* is large? What is the probability that at least one will get his hat back? What is it when *n* is large?

18. Let there are R boxes numbered 1, 2,…, R. At random *n* balls are places in R boxes. What is the probability that exactly *k* balls will be placed in first r (< R) boxes?

19. In a deck of cards there are 52 cards. Out of these 26 cards are red and the rest are black in colour. If you choose 13 cards at random without replacement, what is the probability that you will get 3 red cards?

20. In a deck of cards there are 52 cards with four suits namely club, heart, diamond and spade of equal sizes. If you choose 13 cards at random without replacement then what is the probability that you will get exactly 3 clubs, 4 diamonds, 4 hearts and 2 spades?

21. In a deck of 52 cards, each of the four suits has 13 denominations (Ace, 2,3,4,5,6,7,8,10, Jack, Queen, King). If you choose 4 cards at random with replacement, what will be the probability that you will draw (a) four distinct kings ? (b) a queen each time ?

22. Let n balls be distributed in n numbered boxes so that all $n^n$ arrangements are equally likely (of equal probability). What is the probability that only the $1^{st}$ box will remain empty ?

23. In village of (n+1) people someone originates a rumour. He chooses a person at random and tells it. Second person finds another at random and repeats it and so on … for r (< n) times. (a)What is the probability that the rumour will not come back to the originator? (b) What is the probability that the rumour will not come back to a person who already knew it ?